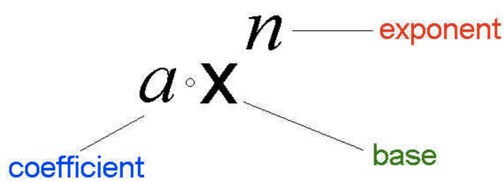
1-2: Exponent Rules



Coefficient: The number in front of the variable

Base: The number/variable being multiplied - attached to the exponent

Exponent (power): how many times the base is multiplied by itself.

Example:

2^3 could be re-written $2 \times 2 \times 2 = 8$

3^5 could be re-written $3 \times 3 \times 3 \times 3 \times 3 = 243$

a^7 could be re-written $a \times a \times a \times a \times a \times a \times a$

******DO NOT confuse 2^3 with 2×3******

They are NOT the same!

Product Rule for exponents

$$a^m \cdot a^n = a^{m+n}$$

Simplify: Watch base/coefficient!

mult. is commutative
$2 \cdot 3 = 3 \cdot 2$

$2^2 \cdot 2^3$

$2^{(2+3)}$

$2^5 = \boxed{32}$

$3z^2 \cdot 4z^4$

$3 \cdot 4 \cdot z^2 \cdot z^4$

$12 \cdot z^{(2+4)}$

$= \boxed{12z^6}$

You try

$(-3)^2 \cdot (-3)^3$

$= (-3)^{(2+3)}$

$= (-3)^5$

$= \boxed{-243}$

$5x^2 \cdot (-2x^5)$

$-10x^{(2+5)}$

$= \boxed{-10x^7}$

Quotient Rule for exponents

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{if } a \neq 0$$

Simplify: Watch base vs coefficient. Write all out and cancel (x/x = 1)

$\frac{11}{11} = 1$

$\frac{103}{103} = 1$

$\frac{x}{x} = 1$

$$\frac{8^5}{8^3} \quad \frac{\cancel{8}\cdot\cancel{8}\cdot\cancel{8}\cdot 8\cdot 8}{\cancel{8}\cdot\cancel{8}\cdot\cancel{8}} = \boxed{8^2}$$

OR

$8^{(5-3)} = \boxed{8^2}$

Same

$$\frac{27z^9}{12z^4}$$

$$\frac{9z^{(9-4)}}{4} = \boxed{\frac{9z^5}{4}}$$

You try

$$\frac{y^8}{y^6} \quad \frac{\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot y\cdot y}{\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}}$$

$\boxed{y^2}$

OR

$y^{(8-6)} = \boxed{y^2}$

$$\frac{-24b^5}{16b^3}$$

$$\frac{-3b^{(5-3)}}{2} = \boxed{\frac{-3b^2}{2}}$$

Zero-exponent Rule

$$a^0 = 1 \qquad \text{if } a \neq 0$$

Simplify

3^0 $\quad$ π^0 $\quad$ $3x^0$ $\quad$ $(\partial\theta + \Phi\Omega - \wp^{\Diamond})^0$

1 $\quad$ 1 $\quad$ $x^0 = 1$, $3 \cdot 1 = \boxed{3}$ $\quad$ 1

Negative-exponent Rule

$$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-n}} = a^n \qquad \text{if } a \neq 0$$

Simplify: only move base, not coefficient!

3^{-4} $\quad$ $4x^{-5}$ $\quad$ $\dfrac{1}{3^{-2}}$

stays

$\dfrac{1}{3^4} = \dfrac{1}{81}$ $\quad$ $\boxed{\dfrac{4}{x^5}}$ $\quad$ $1 \cdot 3^2 = 3^2 = \boxed{9}$

You try

$$5^{-3}$$

$$\frac{1}{5^3} = \boxed{\frac{1}{125}}$$

stays

$$\frac{5}{y^{-3}} = \boxed{5y^3}$$

Simplify

$$\frac{-24b^5}{16b^{-3}}$$

$$\frac{-3b^5 \cdot b^3}{2}$$

$$= \frac{-3b^{(5+3)}}{2} = \boxed{\frac{-3b^8}{2}}$$

$$\frac{50s^2t}{15s^5t^{-4}}$$

$$\frac{10s^{(2-5)}t^1t^4}{3}$$

$$\frac{10s^{-3}t^{(1+4)}}{3}$$

$$\boxed{\frac{10t^5}{3s^3}}$$

How do YOU think we do this?
(hint: write it all the way out!)

$$(3^2)^4$$

$$(3^2)(3^2)(3^2)(3^2)$$

$$\underbrace{3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3}$$

$$3^8$$

Power rule for exponential expressions

$$(a^m)^n = a^{m\cdot n}$$

Simplify

$$(4^3)^5$$

$$4^{3\cdot 5} = 4^{15}$$

$$= \boxed{1073741824}$$

$$[(-3)^3]^2$$

$$(-3)^{(3\cdot 2)}$$

$$= (-3)^6 = \boxed{729}$$

$$(6^3)^0$$

$$6^{(3\cdot 0)}$$

$$6^0 = \boxed{1}$$

You try

$(2^2)^3$

$2^{(2\cdot 3)}$

$2^6 = \boxed{64}$

$(z^3)^{-6}$

$z^{(3\cdot -6)}$

$z^{-18} = \boxed{\frac{1}{z^{18}}}$

$(s^{-3})^{-7}$

$s^{(-3\cdot -7)}$

$\boxed{s^{21}}$

Product to a power

$$(a\cdot b)^n = a^n \cdot b^n$$

Simplify.This case happens with PARENTHESES

$(3z)^4$

$3^4 \cdot z^4$

$\boxed{81z^4}$

$(3y^{-2})^{-3}$

$3^{-3}\cdot y^{(-2\cdot -3)}$

$\frac{1y^6}{3^3}$

$\boxed{\frac{y^6}{27}}$

$(-3a^2)^2$

$(-3)^2 \cdot a^{(2\cdot 2)}$

$\boxed{9a^4}$

You try

$(5y)^3$

$5^3 \cdot y^3$

$\boxed{125y^3}$

$(4a^3)^{-2}$

$4^{-2} \cdot a^{(3 \cdot -2)}$

$\frac{1a^{-6}}{4^2}$

$\boxed{\frac{1}{16a^6}}$

Quotient to a power

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{if } b \neq 0$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad \text{if } a \neq 0,\ b \neq 0$$

Simplify

$$\left(\frac{w}{4}\right)^3 \qquad \frac{w^3}{4^3} = \boxed{\frac{w^3}{64}}$$

$$\left(\frac{2w^2}{y^3}\right)^4 \qquad \frac{(2w^2)^4}{(y^3)^4} \qquad \frac{2^4 w^{(2\cdot 4)}}{y^{(3\cdot 4)}} \qquad \boxed{\frac{16w^8}{y^{12}}}$$

flip

$$\left(\frac{x}{2}\right)^{-5} \qquad \left(\frac{2}{x}\right)^5 \qquad \frac{2^5}{x^5} = \boxed{\frac{32}{x^5}}$$

You try

$$\left(\frac{z}{3}\right)^4 \qquad \frac{z^4}{3^4} = \boxed{\frac{z^4}{81}}$$

flip

$$\left(\frac{4}{3}\right)^{-2} \qquad \left(\frac{3}{4}\right)^2 \qquad \frac{3^2}{4^2} \quad \boxed{\frac{9}{16}}$$

$$\left(\frac{3a^{-2}}{b^4}\right)^3 \qquad \frac{(3a^{-2})^3}{(b^4)^3} \qquad \frac{3a^{(-2\cdot 3)}}{b^{(4\cdot 3)}} \qquad \frac{3a^{-6}}{b^{12}} \qquad \frac{3}{a^6 b^{12}}$$

Rules

$a^0 = 1 \quad \text{if } a \neq 0$

$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-n}} = a^n \quad \text{if } a \neq 0$

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n} \quad \text{if } a \neq 0$

$(a^m)^n = a^{m \cdot n}$

$(a \cdot b)^n = a^n \cdot b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{if } b \neq 0$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad \text{if } a \neq 0,\ b \neq 0$

Simplify

$$\frac{a^3b^{-1}}{(a^2b)^3} \qquad \frac{a^3b^{-1}}{a^{2\cdot 3}\cdot b^3} = \frac{a^3\,b^{-1}}{a^6\,b^3}$$

$(a^2b)^3 = a^{2\cdot 3}\cdot b^3$

$$= \frac{a^3}{a^6b^3b^1} = \frac{\cancel{a}\cdot\cancel{a}\cdot\cancel{a}}{\cancel{a}\cdot\cancel{a}\cdot\cancel{a}\cdot\underbrace{a\cdot a\cdot a}_{3}\cdot\underbrace{b\cdot b\cdot b\cdot b}_{4}}$$

$$= \boxed{\frac{1}{a^3b^4}}$$

Simplify

$$\left(\frac{3xy}{x^2y^{-2}}\right)\cdot\left(\frac{9x^2y^{-3}}{x^3y^2}\right)^{-1}$$

$$\left(\frac{3xy}{x^2y^{-2}}\right)\cdot\left(\frac{x^3y^2}{9x^2y^{-3}}\right)$$

$$\left(\frac{3xy^1\cdot y^2}{x^2}\right)\cdot\left(\frac{x^3y^2\cdot y^3}{9x^2}\right)$$

$$\frac{3x^{(1+3)}\cdot y^{(2+2+3)}}{9x^{(2+2)}} = \frac{3\cancel{x^4}y^7}{9\cancel{x^4}} = \boxed{\frac{y^7}{3}}$$

reduce