1-3: Simplifying Radicals

ex. $\sqrt{4}=2$ $2^2=4$

ex. $\sqrt[3]{8}=2$ $2^3=8$

Definition
nth root

$$\sqrt[n]{b}=a \quad \text{means} \quad b=a^n$$

- if $n\geq 2$ and even then a and b must be greater than or equal to 0.
- if $n\geq 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:

The symbol $\sqrt{\ }$ is called the radical

n is called the index

b is called the radicand

$\sqrt[3]{5}$

3 = index

5 = radicand

Square Roots - Cube Roots: MEMORIZE all in black!!!

$2^2 = 4$	$2^3 = 8$
$3^2 = 9$	$3^3 = 27$
$4^2 = 16$	$4^3 = 64$
$5^2 = 25$	$5^3 = 125$
$6^2 = 36$	$6^3 = 216$
$7^2 = 49$	$7^3 = 343$
$8^2 = 64$	$8^3 = 512$
$9^2 = 81$	$9^3 = 729$
$10^2 = 100$	$10^3 = 1{,}000$
$11^2 = 121$	$11^3 = 1{,}331$
$12^2 = 144$	$12^3 = 1{,}728$

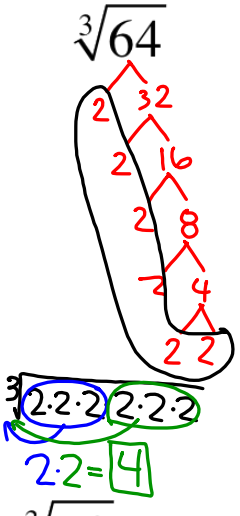
Prime Factorization: Whatever is under the radical, break the number up into all prime numbers.

Prime Numbers: 2, 3, 5, 7, 11, 13, 17, 19, etc.

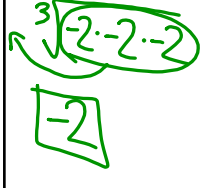
Simplifying: Pull out groups the size of the index.

Evaluate: Pull out groups the size of the index

$\sqrt[3]{64}$

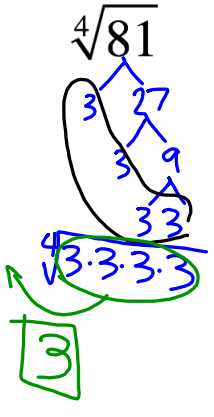


$\sqrt[3]{-8}$



$\sqrt[4]{16}$

$\sqrt[4]{81}$



You try:

$\sqrt[3]{125}$

Shortcut

$5^3 = 125$

$\sqrt[3]{125} = 5$

OR

$\sqrt[3]{125}$

5 25

5 5

$\sqrt[3]{5 \cdot 5 \cdot 5}$

5

$\sqrt[3]{-216}$

Shortcut

$6^3 = 216$

DON'T LOSE THE NEGATIVE!

$\sqrt[3]{-216} = \sqrt[3]{-6 \cdot -6 \cdot -6}$

-6

variable (letter)

Simplifying

If n≥2 is a positive integer and a is a real number, then

$\sqrt[n]{a^n} = a$ if $n \geq 3$ is odd

$\sqrt[n]{a^n} = |a|$ if $n \geq 2$ is even

index

$|a|$ = absolute value

$|-x| = x$

Reduce (remember: even index requires absolute value!)

even

$\sqrt{x^2}$

$\sqrt{x \cdot x}$

$|x|$

abs value

odd

$\sqrt[5]{x^5}$

$\sqrt[5]{x \cdot x \cdot x \cdot x \cdot x}$

x

You try!

odd

$\sqrt[3]{x^3}$

x

even

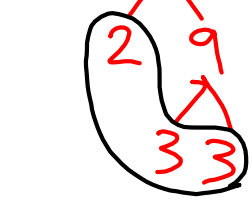
$\sqrt[6]{z^6}$

$|z|$

abs value

Simplify: Remember, can only pull out groups the size of the index. All else must stay UNDER the radical.

$\sqrt{18}$

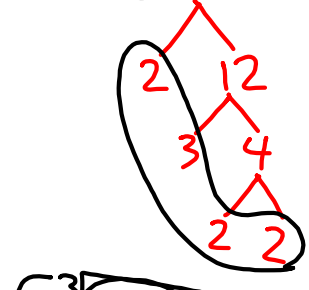


$\sqrt{2 \cdot 3 \cdot 3}$

$3\sqrt{2}$

Simplify

$5\sqrt[3]{24}$

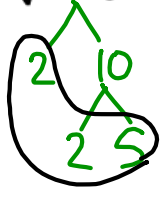


$5\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$

$2 \cdot 5 \cdot \sqrt{3}$

$10\sqrt{3}$

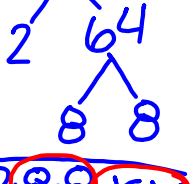
$\sqrt[4]{20}$



no groups of 4-already simplified

(remember $\sqrt{x^2} = |x|$)

even $\sqrt{128x^2}$

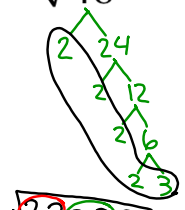


$\sqrt{2 \cdot 8 \cdot 8 \cdot x \cdot x}$

$8|x|\sqrt{2}$

You try!

$\sqrt{48}$



$\sqrt{2\cdot2\cdot2\cdot3}$

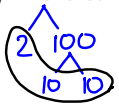
$2\cdot2\cdot\sqrt{3} = \boxed{4\sqrt{3}}$

$4\sqrt[3]{54}$

$4\sqrt[3]{2\cdot3\cdot3\cdot3}$

$3\cdot4\sqrt[3]{2} = \boxed{12\sqrt[3]{2}}$

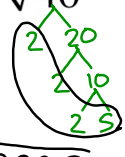
$\sqrt[2]{200a^2}$ even



$\sqrt{2\cdot10\cdot10\cdot a\cdot a}$

$\boxed{10|a|\sqrt{2}}$

$\sqrt[4]{40}$



$\sqrt[4]{2\cdot2\cdot2\cdot5}$

no groups of 4, already simplified

$\boxed{\sqrt[4]{40}}$

Simplify

even!

$\sqrt[2]{12p^2q}$

$\sqrt{2\cdot2\cdot3\cdot p\cdot p\cdot q}$

$\boxed{2|p|\sqrt{3q}}$

Remember that

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

For example

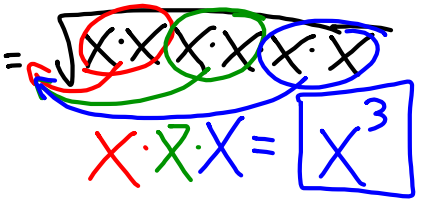
$$\sqrt{x^2} = |x| \qquad \sqrt[3]{x^3} = x \qquad \sqrt[4]{x^4} = |x| \qquad \text{and so on}$$

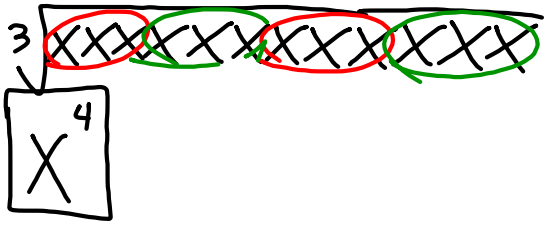
But to make our life easier some instructions will say "Assume all variables are greater then or equal to zero." In which case:

$$\sqrt{x^2} = x \qquad \sqrt[3]{x^3} = x \qquad \sqrt[4]{x^4} = x \qquad \text{and so on}$$

SO READ YOUR INSTRUCTIONS!!!

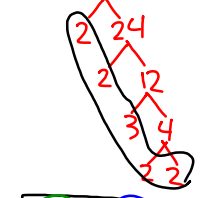
Reduce, assuming all variables are greater than or equal to zero.

$\sqrt{x^6} =$ 

$\sqrt[3]{x^{12}} =$ 

You try! Assume all variables are greater than or equal to zero

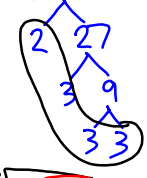
$\sqrt{48}$



$\sqrt{2\cdot2\cdot3}\,2\cdot2$

$2\cdot2\cdot\sqrt{3} = \boxed{4\sqrt{3}}$

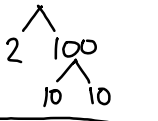
$4\sqrt[3]{54}$



$4\sqrt[3]{2\cdot3\cdot3\cdot3}$

$3\cdot4\sqrt[3]{2} = \boxed{12\sqrt[3]{2}}$

$\sqrt{200a^2}$



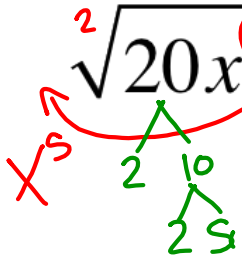
$\sqrt{2\cdot10\cdot10\cdot a\cdot a}$

$\boxed{10a\sqrt{2}}$

Reduce, assuming all variables are greater than or equal to zero.

$10/2 = 5$

$\sqrt[2]{20x^{10}}$



$X^5\sqrt{2\cdot2\cdot5}$

$\boxed{2X^5\sqrt{5}}$

You try! Assume all variables are greater than or equal to zero.

$\sqrt[2]{75a^6}$

$6/2 = 3$

a^3 s is s s

$a^3\sqrt{5\cdot5\cdot5}$

$5a^3\sqrt{5}$

Simplify! not assuming! check abs value!

even $\sqrt[2]{80a^3}$

4 20 4 5

$\sqrt{4\cdot4\cdot5\cdot a\cdot a\cdot a}$

$4|a|\sqrt{5a}$

odd $\sqrt[3]{27m^4n^{14}}$

$3^3 m^3 \cdot m \cdot n^{12} \cdot n^2$

$3\ m\ n^4$

$3mn^4\sqrt{mn^2}$

You Try!

$\sqrt[3]{128x^6y^{10}}$

$128 = 2 \cdot 64$, $64 = 2 \cdot 32$, $32 = 2 \cdot 16$, $16 = 2 \cdot 8$, $8 = 2^3$

$x^6 = x^3 \cdot x^3$

$y^{10} = y^9 \cdot y$

$\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x^3 \cdot x^3 \cdot y^3 \cdot y^3 \cdot y^3 \cdot y}$

$2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y \sqrt{2y}$

$\boxed{4x^2y^3\sqrt{2y}}$