1-4 Rational Exponents

Rational exponent Calc task

1, 4, 9, 16, 25, 36

$a^{\left(\frac{1}{2}\right)} =$ _____

1, 8, 27, 64, 125, 216

$$a^{\left(\frac{1}{3}\right)} = \_\_\_\_\_$$

1, 16, 81, 256, 625, 1296

$$a^{\left(\frac{3}{4}\right)} = \_\_\_\_\_$$

$$a^{\left(\frac{m}{n}\right)} = \_\_\_\_\_$$

Fractional exponent

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

n is an integer bigger than or equal to 2

Write each of the following as a radical and simplify, if possible.

$9^{\frac{1}{2}}$

$\sqrt[2]{9}$

$\sqrt[2]{3 \cdot 3}$

3

$(-64)^{\frac{1}{3}}$

$\sqrt[3]{-64}$

$\sqrt[3]{-4 \cdot -4 \cdot -4}$

-4

$100^{\frac{1}{2}}$

$\sqrt[2]{100}$

$\sqrt[2]{10 \cdot 10}$

10

$-100^{\frac{1}{2}}$

$-\sqrt[2]{100}$

$-\sqrt[2]{10 \cdot 10}$

-10

$z^{\frac{1}{2}}$

$\sqrt{z}$

You try

$25^{\frac{1}{2}}$

$\sqrt{25} = \sqrt{5 \cdot 5}$

5

$(-27)^{\frac{1}{3}}$

$\sqrt[3]{-3 \cdot -3 \cdot -3}$

-3

$-64^{\frac{1}{2}}$

$-\sqrt{64}$

$-\sqrt{8 \cdot 8}$

-8

$b^{\frac{1}{2}}$

$\sqrt{b}$

Rewrite in exponent form

$\sqrt[4]{7a}$

$(7a)^{\frac{1}{4}}$

$\sqrt[5]{\frac{xy^3}{4}}$ $\left(\frac{xy^3}{4}\right)^{\frac{1}{5}}$

You try

$\sqrt[5]{8b}$

$(8b)^{\frac{1}{5}}$

$\sqrt[8]{\frac{mn^5}{3}}$ $\left(\frac{mn^5}{3}\right)^{\frac{1}{8}}$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

a is real, m/n is a rational number in lowest terms with n bigger or equal to 2

Write each of the following as a radical and simplify, if possible.

$25^{\frac{3}{2}}$

$\left(\sqrt{25}\right)^3$

$(5)^3 = 125$

$64^{\frac{2}{3}}$

$\left(\sqrt[3]{64}\right)^2$

$(4)^2 = 16$

$-9^{\frac{5}{2}}$

$-\left(\sqrt{9}\right)^5$

$-(3)^5 = -243$

$(-8)^{\frac{4}{3}}$

$\left(\sqrt[3]{-8}\right)^4$

$(-2)^4 = 16$

$-(81)^{\frac{7}{2}}$

$-\left(\sqrt{81}\right)^7$

$-(9)^7$

$-4,782,969$

You try

$27^{\frac{2}{3}}$

$(\sqrt[3]{27})^2$

$(3)^2 = 9$

$16^{\frac{3}{2}}$

$(\sqrt{16})^3$

$(4)^3 = 64$

$(-25)^{\frac{5}{2}}$

$-(\sqrt{25})^5$

$-(5)^5 = -3125$

$-16^{\frac{3}{4}}$

$-(\sqrt[4]{16})^3$

$16 = 2 \cdot 8$, $8 = 2 \cdot 4$, $4 = 2 \cdot 2$

$-(\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2})^3$

$-(2)^3$

-8

Rewrite in exponent form

$\sqrt[3]{x^2}$

$(x^2)^{\frac{1}{3}}$

$x^{\frac{2}{1} \cdot \frac{1}{3}}$

$x^{\frac{2}{3}}$

$(\sqrt[5]{10a^2b})^4$

$(10a^2b)^{\frac{4}{5}}$

$10^{\frac{4}{5}} a^{\frac{2}{1} \cdot \frac{4}{5}} b^{\frac{4}{5}}$

$10^{\frac{4}{5}} a^{\frac{8}{5}} b^{\frac{4}{5}}$

You try

$\sqrt[8]{a^3}$

$(a^{\frac{3}{1}})^{\frac{1}{8}}$

$a^{\frac{3}{8}}$

$\left(\sqrt[4]{12ab^3}\right)^9$

$(12ab^3)^{\frac{9}{4}}$

$12^{\frac{9}{4}}a^{\frac{9}{4}}b^{\frac{3}{1}\cdot\frac{9}{4}}$

$12^{\frac{9}{4}}a^{\frac{9}{4}}b^{\frac{27}{4}}$

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} \quad \text{and} \quad \frac{1}{a^{-\frac{m}{n}}} = a^{\frac{m}{n}}$$

$\frac{m}{n}$ is a rational number, and a is a nonzero real number

Write each of the following as a radical and simplify, if possible.

$36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{\sqrt{36}} = \boxed{\frac{1}{6}}$

$\frac{1}{27^{-\frac{2}{3}}} = 27^{2/3} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = \boxed{9}$

$(6a)^{-\frac{5}{4}} = \frac{1}{(6a)^{\frac{5}{4}}} = \boxed{\frac{1}{\left(\sqrt[4]{6a}\right)^5}}$

You try

$81^{-\frac{1}{2}} = \frac{1}{81^{1/2}} = \frac{1}{\sqrt{81}} = \boxed{\frac{1}{9}}$

$\frac{1}{8^{-\frac{2}{3}}} = 8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = \boxed{4}$

$(13x)^{-\frac{3}{2}} = \frac{1}{(13x)^{3/2}} = \boxed{\frac{1}{\left(\sqrt{13x}\right)^3}}$

Simplify Expressions Using the Laws of Exponents

Just a reminder:

Rules

$a^0 = 1 \quad \text{if } a \neq 0$

$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-n}} = a^n \quad \text{if } a \neq 0$

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n} \quad \text{if } a \neq 0$

$(a^m)^n = a^{m \cdot n}$

$(a \cdot b)^n = a^n \cdot b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{if } b \neq 0$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad \text{if } a \neq 0,\ b \neq 0$

After you simplify you should have:

- Only positive exponents.
- Each base only occurring once.
- Have no parentheses in the expression.
- No powers written to powers.

Simplify each of the following:

$3 \cdot \frac{1}{2} = \frac{3}{6}$

$27^{\frac{1}{2}} \cdot 27^{\frac{5}{6}}$

$27^{\frac{1}{2}+\frac{5}{6}} = 27^{\frac{3}{6}+\frac{5}{6}} = 27^{\frac{8}{6}} = 27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4$

$= (3)^4 = \boxed{81}$

$\frac{8^{\frac{1}{3}}}{8^{\frac{5}{3}}} = 8^{\frac{1}{3}-\frac{5}{3}} = 8^{-\frac{4}{3}} = \frac{1}{8^{4/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^4}$

$= \frac{1}{(2)^4} = \boxed{\frac{1}{16}}$

You Try

LCD of 4 & 6 = 12

$3 \cdot 3 = 9$, $3 \cdot 4 = 12$, so $\frac{3}{4} = \frac{9}{12}$

$2 \cdot 1 = 2$, $2 \cdot 6 = 12$, so $\frac{1}{6} = \frac{2}{12}$

$5^{\frac{3}{4}} \cdot 5^{\frac{1}{6}}$

$5^{\frac{9}{12}} \cdot 5^{\frac{2}{12}} = 5^{\frac{9}{12}+\frac{2}{12}} = \boxed{5^{11/12}}$ check $\left(\sqrt[12]{5}\right)^{11}$

$\frac{32^{\frac{6}{5}}}{32^{\frac{3}{5}}} = 32^{\frac{6}{5}-\frac{3}{5}} = 32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3$

$= (2)^3 = \boxed{8}$

Simplify each of the following:

$$\left(36^{\frac{2}{5}}\right)^{\frac{5}{4}} = 36^{\frac{2\cdot5}{5\cdot4}} = 36^{\frac{10}{20}} = 36^{\frac{1}{2}} = \sqrt{36} = \boxed{6}$$

$$\left(x^{\frac{1}{2}} \cdot y^{\frac{2}{3}}\right)^{\frac{3}{2}} = x^{\frac{1}{2}\cdot\frac{3}{2}} \cdot y^{\frac{2}{3}\cdot\frac{3}{2}} = x^{\frac{3}{4}} \cdot y^{\frac{6}{6}}$$

$$= \boxed{x^{\frac{3}{4}} \cdot y} \quad \text{check } (\sqrt[4]{x})^3 \cdot y$$

You Try

$$\left(100^{\frac{3}{8}}\right)^{\frac{4}{3}} = 100^{\frac{3\cdot4}{8\cdot3}} = 100^{\frac{4}{8}} = 100^{\frac{1}{2}} = \sqrt{100}$$

$$= \boxed{10}$$

$$\left(a^{\frac{3}{2}} \cdot b^{\frac{5}{4}}\right)^{\frac{2}{3}} = a^{\frac{3}{2}\cdot\frac{2}{3}} \cdot b^{\frac{5}{4}\cdot\frac{2}{3}} = a^{\frac{6}{6}} \cdot b^{\frac{10}{12}}$$

$$= \boxed{a \cdot b^{\frac{5}{6}}}$$

Simplify the following:

$$\left(x^{\frac{2}{3}}y^{-1}\right)\cdot\left(x^{-1}y^{\frac{1}{2}}\right)^{\frac{2}{3}}$$

$$x^{\frac{2}{3}}y^{-1}\cdot x^{-\frac{1}{1}\cdot\frac{2}{3}}\cdot y^{\frac{1}{2}\cdot\frac{2}{3}}$$

$$x^{\frac{2}{3}}y^{-1}\cdot x^{-\frac{2}{3}}\cdot y^{\frac{1}{3}}$$

$$x^{\frac{2}{3}}\cdot x^{-\frac{2}{3}}\cdot y^{-1}\cdot y^{\frac{1}{3}} \qquad -1=\frac{-1}{1}\cdot\frac{3}{3}=\frac{-3}{3}$$

$$x^{\frac{2}{3}-\frac{2}{3}}\cdot y^{-\frac{3}{3}+\frac{1}{3}}$$

$$x^{0}\cdot y^{-2/3}=\frac{1}{y^{2/3}}$$

($x^0=1$)

check

$$\frac{1}{\left(\sqrt[3]{y}\right)^2}$$

Use rational exponents to simplify the radicals.

$$\sqrt[8]{16^4}$$

$$\left(16^{\frac{4}{1}}\right)^{\frac{1}{8}}$$

$$16^{4/8}=16^{1/2}$$

$$=\sqrt{16}=4$$

$$\sqrt[3]{64x^6y^3}$$

$$\left(64x^6y^3\right)^{\frac{1}{3}}$$

$$64^{\frac{1}{3}}x^{\frac{6}{1}\cdot\frac{1}{3}}\cdot y^{\frac{3}{1}\cdot\frac{1}{3}}$$

$$\sqrt[3]{64}\cdot x^2\cdot y$$

$$4x^2y$$

Use rational exponents to simplify the radicals.

$$\frac{\sqrt{x}}{\sqrt[3]{x^2}}$$

LCD $2, 3 = 6$

$\frac{3}{3}\cdot\frac{1}{2} = \frac{3}{6}$ $\quad \frac{2}{2}\cdot\frac{2}{3} = \frac{4}{6}$

$$\frac{x^{\frac{1}{2}}}{x^{\frac{2}{3}}} = x^{\frac{1}{2}-\frac{2}{3}} = x^{\frac{3}{6}-\frac{4}{6}} = x^{-\frac{1}{6}} = \frac{1}{x^{1/6}}$$

OR $\frac{1}{\sqrt[6]{x}}$

$$\sqrt{\sqrt[3]{z}}$$

$$\sqrt{z^{1/3}}$$

$$z^{\frac{1}{3}\cdot\frac{1}{2}}$$

$$z^{1/6} \text{ or } \sqrt[6]{z}$$