2-2: Piecewise Functions

I would like you to make up your own story that matches the graph below:

Snow drift

Large rock

Hiding place

Time
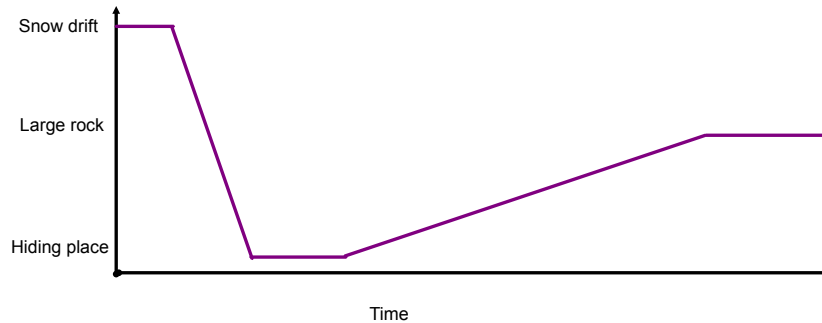
Here's my story: A penguin was sleeping in a snowdrift when it was awakened by a polar bear. It ran quickly to a hiding place where it waited for the bear to pass by. Then it walked slowly back towards the snowdrift. The bear jumped out from behind a large rock and ate him before he got there.

I would like for you to draw a position vs. time graph of your day. You start at home then come (very quickly because you're late) to school early in the morning and stay here for 7 hours. You then go to your friends' house and stay there for 2 hours. You then walk slowly home.
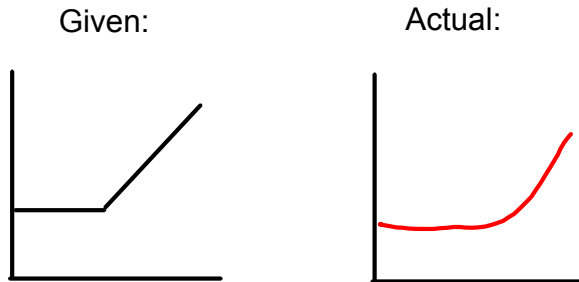
school

friend's house

home

time

Motion Detector Task!

(far)

Distance
from MD

Time

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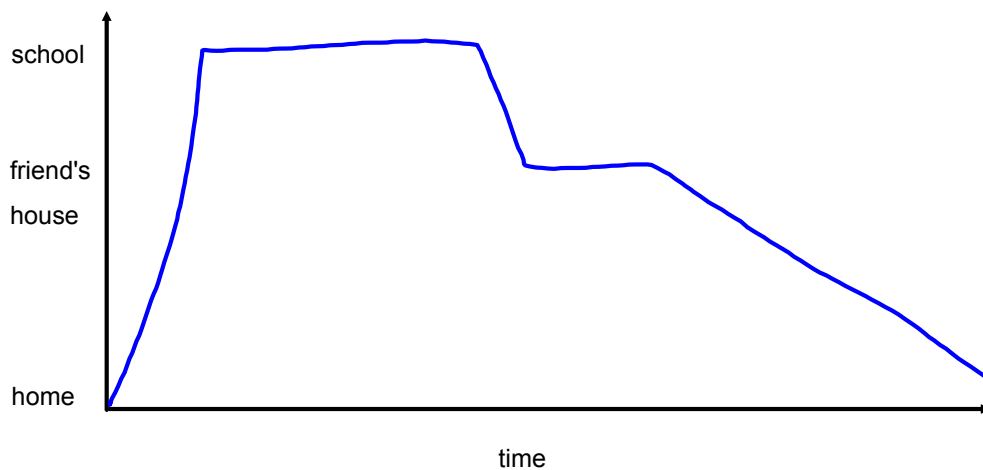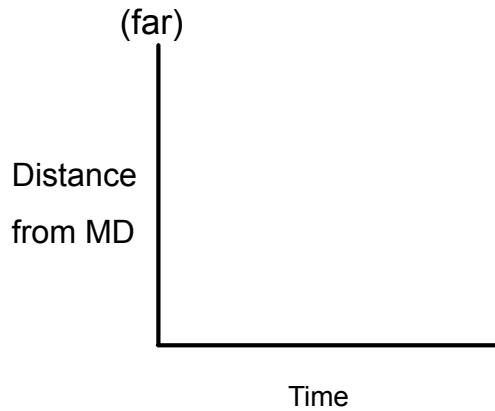
1. Prediction: Start far away, walk towards MD
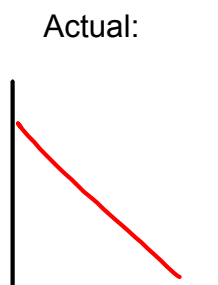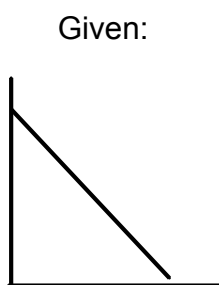
Given:

Actual:

2. Prediction: Stand still, then start walking away
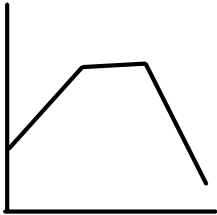
Given:

Actual:

3. Prediction: Walk towards it quickly, then walk slowly away.
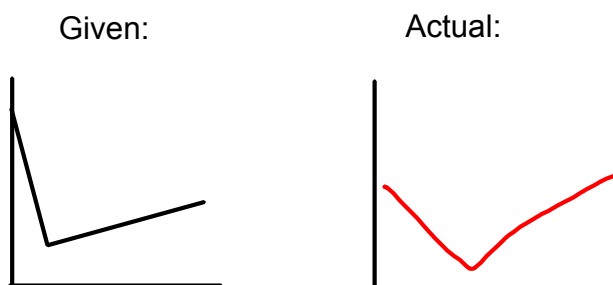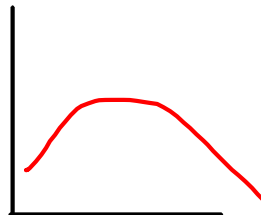
Given:

Actual:

4. Prediction: Start half way, walk away, stand still, then quickly walks towards
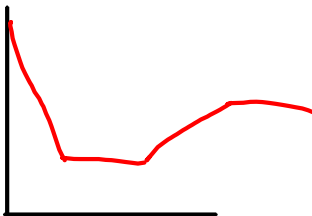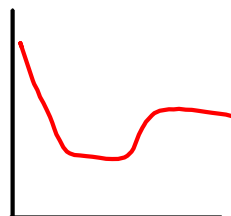
Given:

Actual:

Bob started far away. Then he quickly ran towards the detector. Then he stopped. Then he walked away slowly. Then he stopped.

Prediction:

Actual:

Piecewise Function: A function whose definition changes depending on the value of the independent variable.

Recall: A function must pass the *vertical line test* (meaning two points cannot intersect a given vertical line at any point of the graph)

Example of what piecewise might look like:
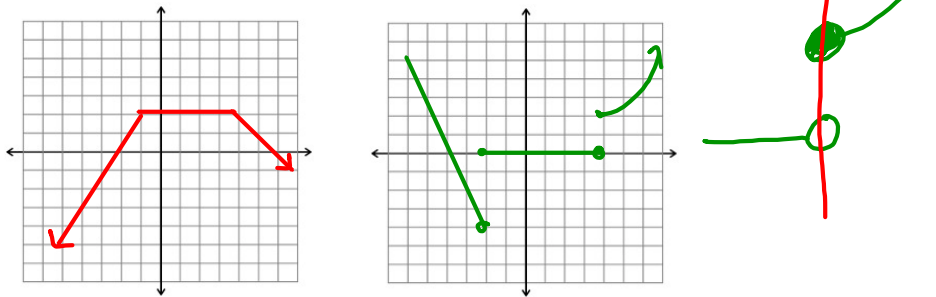
Graphing a piecewise

2 points per piece (endpoints of interval)

< , > open circles ○

≤ , ≥ closed circles ●

or arrow →

(2,-3)
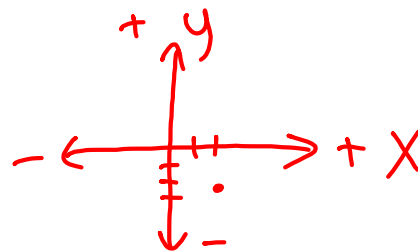
Piece	(x	,	y)	open/closed

endpoint of interval   x plugged in =_____

Then graph the points and connect the pieces!!!

$$f(x) = \begin{cases} x+6, & x \le -3 \\ 3x+4, & -3 < x < 1 \end{cases}$$

Piece	(x , y)	open/closed
$x+6$	$(-3, 3)$	●
	$(-4, 2)$	→
$3x+4$	$(-3, -5)$	○
	$(1, 7)$	○

Graph

$$f(x) = \begin{cases} 5, & -4 < x < -3 \\ -x, & -3 \le x < 1 \\ 2x-3, & 1 \le x \le 4 \end{cases}$$

Piece	(x , y)	open/closed
5	$(-4, 5)$	○
	$(-3, 5)$	○
$-x$	$(-3, 3)$	●
	$(1, -1)$	○
$2x-3$	$(1, -1)$	●
	$(4, 5)$	●
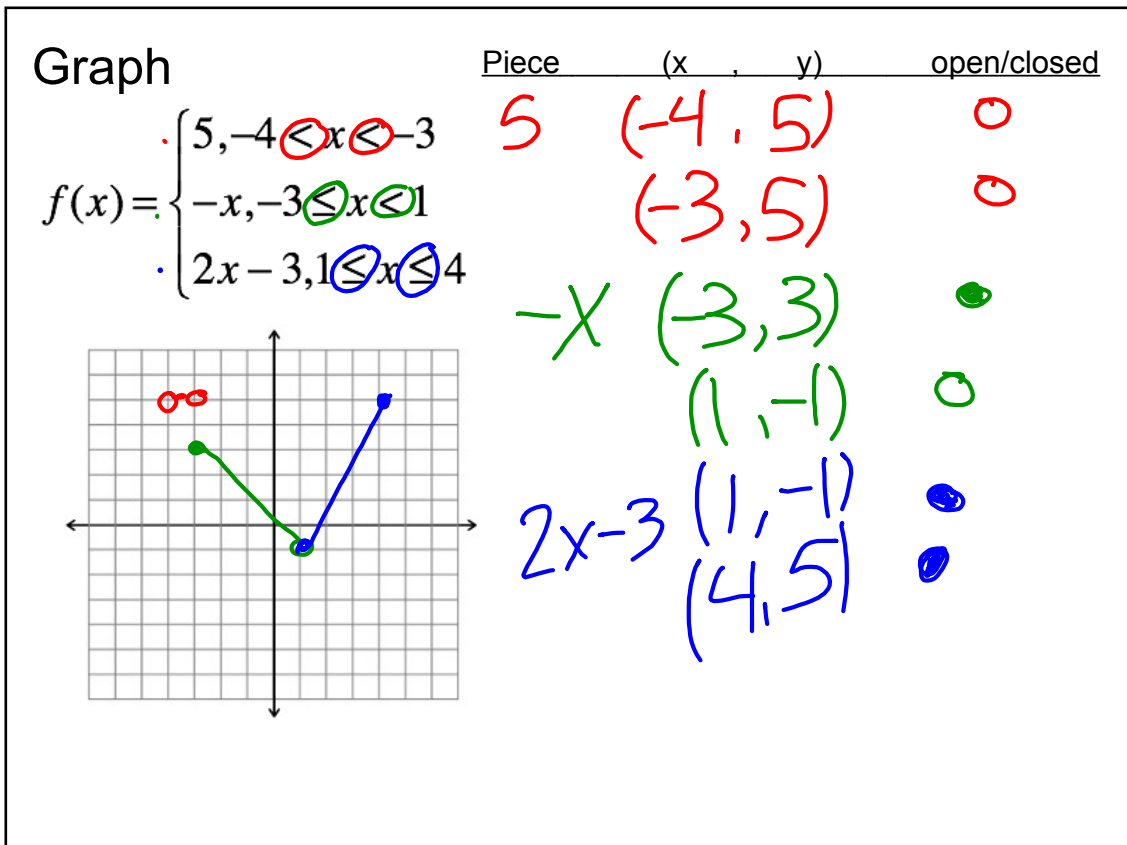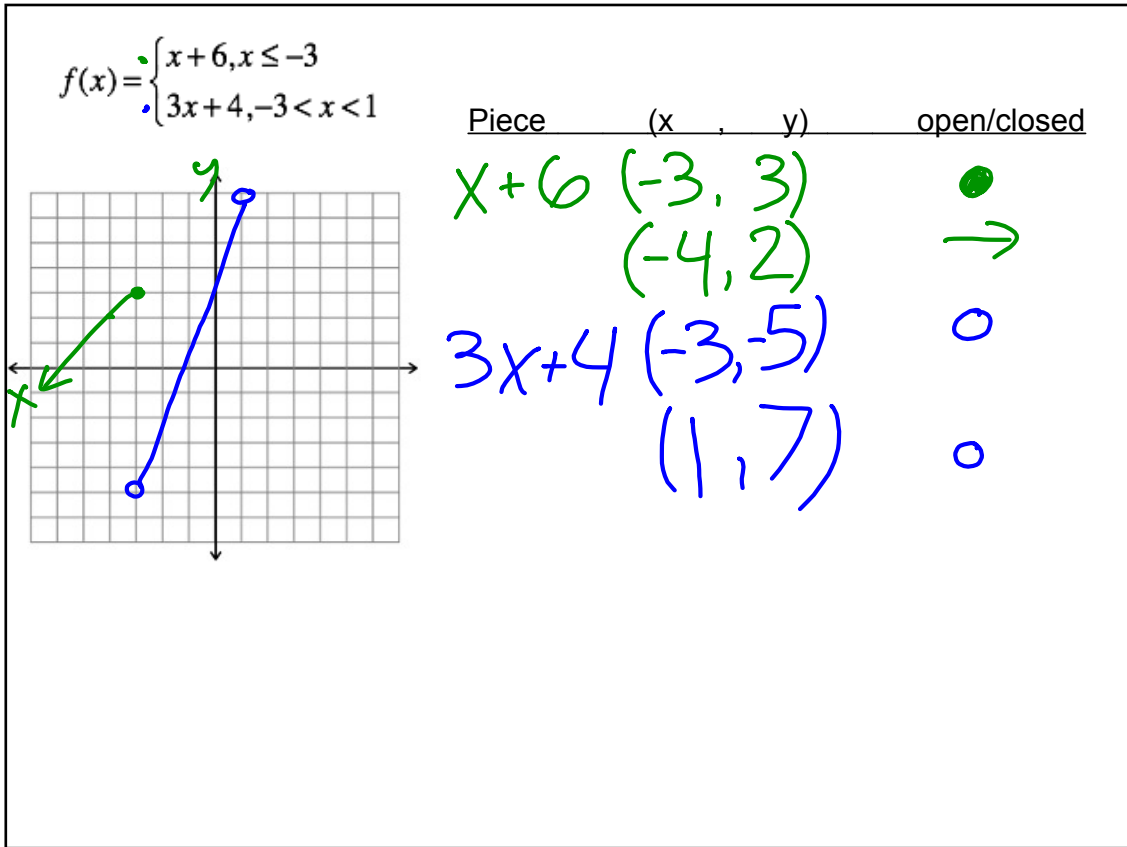
You Try:

$$f(x) = \begin{cases} x+1, & -5 < x \le 1 \\ 3, & 1 < x < 4 \\ 2x-3, & x \ge 4 \end{cases}$$

Piece	(x , y)	open/closed
X+1	(-5, -4)	O
	(1, 2)	●
3	(1, 3)	O
	(4, 3)	O
2x-3	(4, 5)	●
	(5, 7)	→

Remember:

slope = m = $\dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equations of a line:

point   slope

point-slope form: y - y$_1$ = m(x - x$_1$)   then solve for y

slope-intercept form: y = mx + b

slope   y-intercept

Find the equation of the line given the graph/ points:

$x_1 \, y_1 \quad x_2 \, y_2$

(2 ,4) and (6,-8)

$$\frac{-8-4}{6-2} = \frac{-12}{4} = \boxed{-3}$$

$y - 4 = -3(x - 2)$

$y - 4 = -3x + 6$

$\quad +4 \qquad\qquad +4$

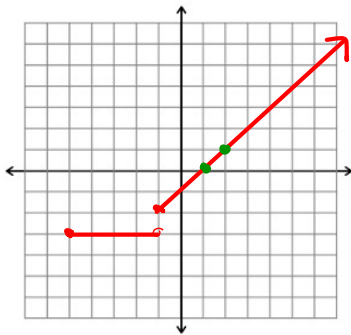$\boxed{y = -3x + 10}$

$m = \dfrac{3}{-1} = -3$

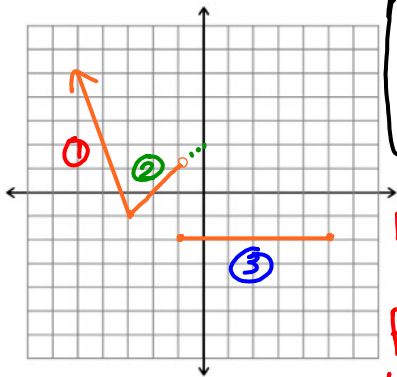$y = mx + b$

$\boxed{y = -3x - 3}$
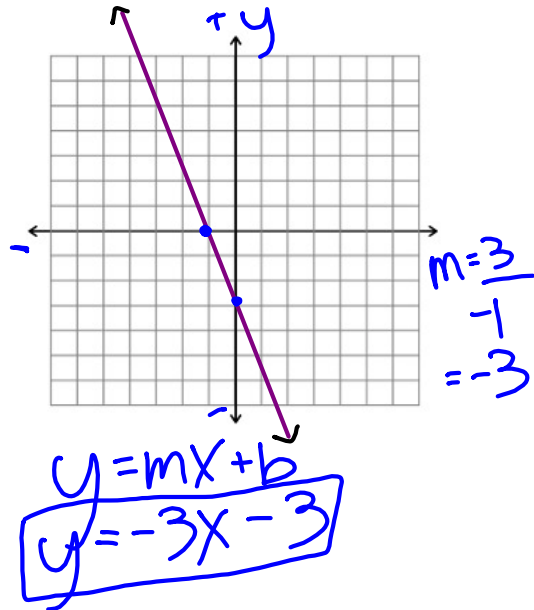
Working Backwards!

1. Find the equation of each line (segment)

2. Determine the domain of each piece (don't forget to watch for open circles <, > or closed circles ≤

3. Write as a piecewise function

$$f(x) = \begin{cases} -3, & -5 \le x < -1 \\ x - 1, & x \ge -1 \end{cases}$$

$y = mx + b$

$y = x - 1$

Write the equation for the piecewise function:

$$f(x) = \begin{cases} -3x-10, & x \le -3 \\ x+2, & -3 < x < -1 \\ -2, & -1 \le x \le 5 \end{cases}$$

$$m = \frac{rise}{run} = \frac{3}{-1} = -3$$

Point $(-3, -1)$ [x₁, y₁ labeled as x_1, y_1]

$y - -1 = -3(x - -3)$

$y + 1 = -3(x+3)$

$y + 1 = -3x - 9$

$-1 \qquad -1$

$$y = -3x - 10$$

$y = mx + b$

slope   y-int

$y = x + 2$

Distance from home (mi)

Time (h)
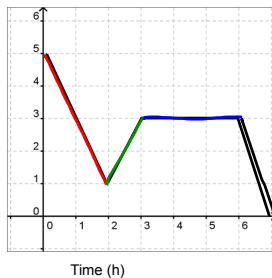
1. What is happening at time=2 h?

turned around

2. What is happening from time=0 h until time=2 h? What is the equation of this line?

heading home

$y = -2x + 5$

3. What is happening from time=2 h until time=3 h? What is the equation of this line? going away from home

$y = 2x - 3$

4. What is happening from time=3h to time=6h? What is the equation of this line? didn't move

$y = 3$

5. What is happening from time=6h to time=7h. What is the equation of this line? heading home

$y = -3x + 21$

6. Write and equation for this graph. Be sure to include the domain!!!

$$f(x) = \begin{cases} 2x+5, & 0 \le x < 2 \\ 2x-3, & 2 \le x < 3 \\ 3, & 3 \le x < 6 \\ -3x+21, & 6 \le x < 7 \end{cases}$$

5. a. turned around

   b. going at a constant rate
   (away from train station)

$(0, 0)$ $(40, 250)$

x_1, y_1 x_2, y_2

$$\frac{250 - 0}{40 - 0} = \frac{250}{40} = \boxed{\frac{25}{4}}$$