

## 2-4 Step Functions (16.2 in book)

pgs. 1146-1150 in student book

### Objectives:

- I can write and graph step function problem situations.
- I can analyze the graphs of step functions.
- I can use a calculator to graph a step function.

pg. 1146

### Problem 2: Taxi Fares

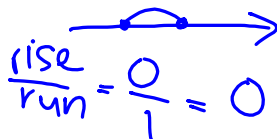
In 2006, the rate for a taxi ride in Macon, Georgia, was \$1.20 for the first mile or part of a mile, and \$1.20 for each additional mile or part of a mile.

$x = \text{miles}$        $y = \text{cost}$

- Define a piecewise function,  $g(x)$ , for the cost of a taxi ride up to 5 miles.

$$f(x) = \begin{cases} 1.20 & 0 < x \leq 1 \\ 2.40 & 1 < x \leq 2 \\ 3.60 & 2 < x \leq 3 \\ 4.80 & 3 < x \leq 4 \\ 6.00 & 4 < x \leq 5 \end{cases}$$

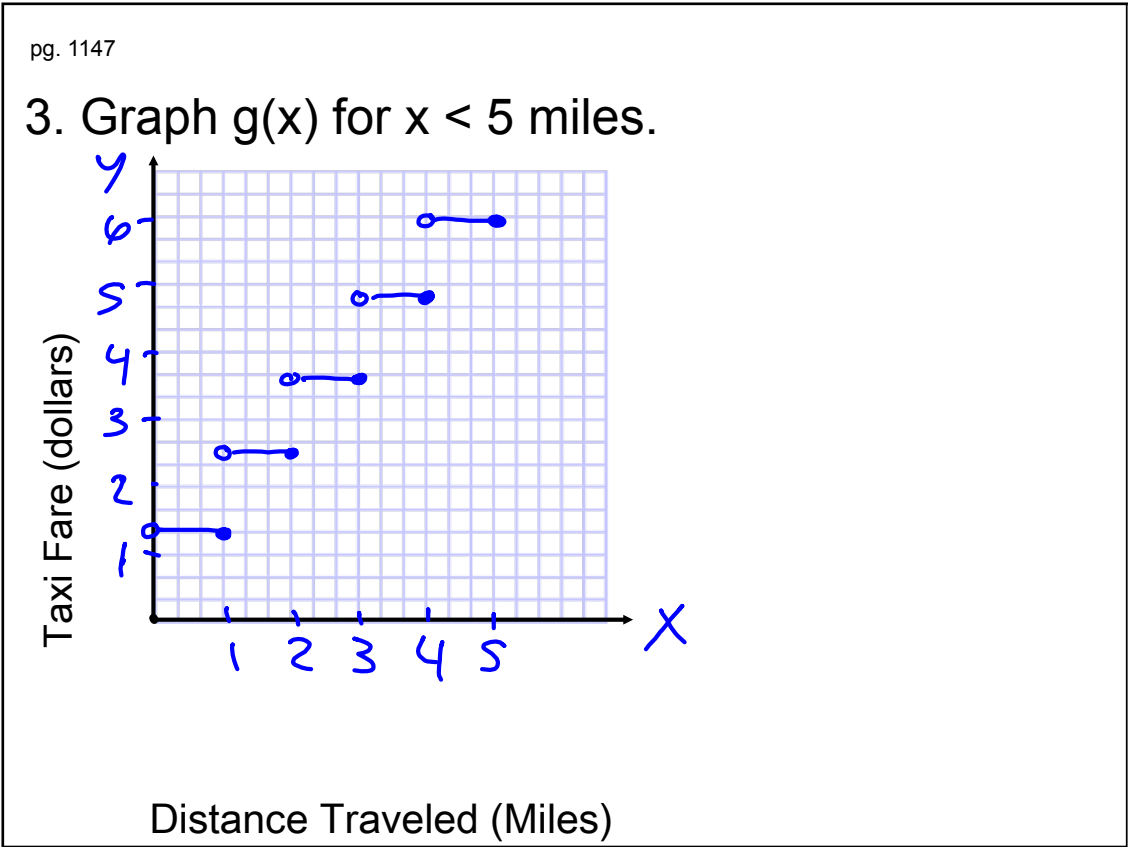
- What is the slope of each interval? Explain your reasoning.



$$\frac{\text{rise}}{\text{run}} = \frac{0}{1} = 0$$

$m = 0$   
constant

Piece	(x, y)	o ●
1.20	0, 1.20	o
	1, 1.20	●
2.40	1, 2.40	o
	2, 2.40	●
3.60	2, 3.60	o
	3, 3.60	●
4.80	3, 4.80	o
	4, 4.80	●
6	4, 6	o
	5, 6	●



pg. 1147

You have just graphed a *step function*.

A **step function** is a piecewise function whose pieces are disjoint <sup>\*</sup>  
constant functions. \*

5. Why do you think this function is called a step function?

that's what  
it looks like!

\*Calculator steps in book.

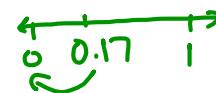
pg. 1148


### Problem 3 Special Step Functions

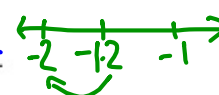
The *greatest integer function* is a special kind of step function. The **greatest integer function**, also known as the **floor function**,  $G(x) = \lfloor x \rfloor$  is defined as the greatest integer less than or equal to  $x$ .

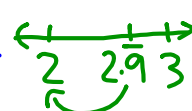
1. Evaluate each using the greatest integer function.

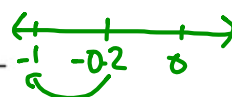
a.  $\lfloor 2 \rfloor = \underline{2}$

b.  $\lfloor 0.17 \rfloor = \underline{0}$   A number line from 0 to 1 with a tick mark at 0.17. A green arrow points from 0.17 to 0, and a green bracket is drawn below the line from 0 to 0.17.

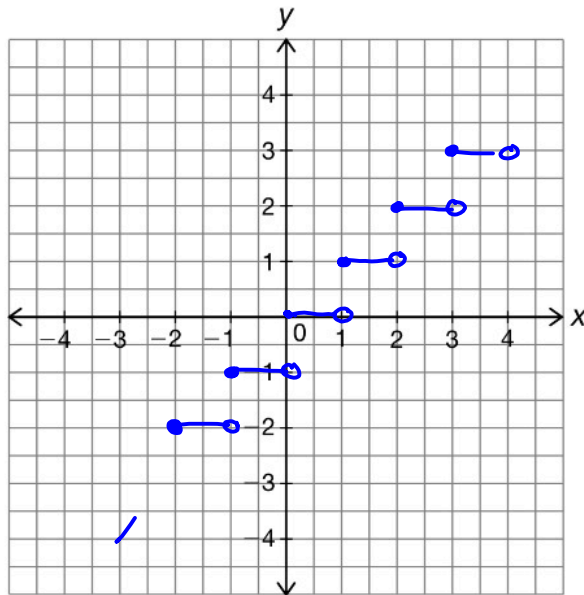
c.  $\lfloor 2.34 \rfloor = \underline{2}$   A number line from 2 to 3 with a tick mark at 2.34. A green arrow points from 2.34 to 2, and a green bracket is drawn below the line from 2 to 2.34.

d.  $\lfloor -1.2 \rfloor = \underline{-2}$   A number line from -2 to 0 with tick marks at -2, -1.2, and -1. A green arrow points from -1.2 to -2, and a green bracket is drawn below the line from -2 to -1.2.

e.  $\lfloor 2.99999 \rfloor = \underline{2}$   A number line from 2 to 3 with a tick mark at 2.99999. A green arrow points from 2.99999 to 2, and a green bracket is drawn below the line from 2 to 2.99999.

f.  $\lfloor -0.2 \rfloor = \underline{-1}$   A number line from -1 to 0 with tick marks at -1, -0.2, and 0. A green arrow points from -0.2 to -1, and a green bracket is drawn below the line from -1 to -0.2.

pg. 1148

2. Graph  $G(x) = \lfloor x \rfloor$ .

Points

$(0, 0)$

$(0.1, 0)$

$(0.9, 0)$

$(1, 1)$

$(1.1, 1)$

$(1.9, 1)$

$(2, 2)$

pg. 1149

The *least integer function* is another special kind of step function. The **least integer function**  $L(x) = \lceil x \rceil$  also known as the **ceiling function**, is defined as the least integer greater than or equal to  $x$ .

3. Calculate each:

a.  $\lceil 2 \rceil = \underline{2}$

b.  $\lceil 0.17 \rceil = \underline{1}$

c.  $\lceil 2.34 \rceil = \underline{3}$

d.  $\lceil -1.2 \rceil = \underline{-1}$

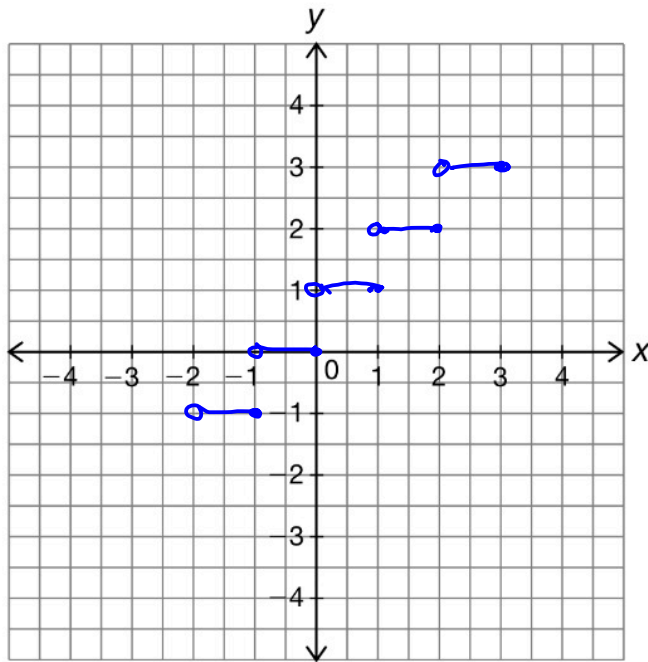
*(Handwritten diagram: A number line from -2 to 0. A red arrow points from -1.2 to -1, with a bracket underneath labeled -1.)*

e.  $\lceil 2.99999 \rceil = \underline{3}$

f.  $\lceil -0.2 \rceil = \underline{0}$

*(Handwritten diagram: A number line from -1 to 0. A red arrow points from -0.2 to 0, with a bracket underneath labeled 0.)*

pg. 1149

4. Graph  $L(x) = \lceil x \rceil$ .

$(0, 0)$   
 $(0.1, 1)$   
 $(0.9, 1)$   
 $(1, 1)$   
 $(1.1, 2)$   
 $(1.9, 2)$   
 $(2, 2)$

6.  $x = \$ \text{ spent}$   
 $y = \text{value of coupons}$

interval  
equation/piece

$$f(x) = \begin{cases} 0 & 0 \leq x < 20 \\ 2 & 20 \leq x < 40 \\ 4 & 40 \leq x < 60 \\ 6 & 60 \leq x < 80 \\ 8 & 80 \leq x < 100 \end{cases}$$

7.  $x = \text{sales} \rightarrow \text{interval}$   
 $y = \$ \text{ returned} \rightarrow \text{piece}$

$$f(x) = \begin{cases} 100, & 0 < x \leq 250 \\ 225, & 250 < x \leq 500 \\ 350, & 500 < x \leq 750 \\ 475, & 750 < x \leq 1000 \end{cases}$$

14.  $x = \text{weight}$   
 $y = \text{cost}$

$$f(x) = \begin{cases} 5, & 0 < x \leq 10 \\ 10, & 10 < x \leq 20 \\ 15, & 20 < x \leq 30 \\ 20, & 30 < x \leq 40 \\ 25, & 40 < x \leq 50 \end{cases}$$