Unit 3 Review
Secondary III

Name: KEY
Date: ______________ Class: ________

1. Are (x+2) and (x-6) factors of $f(x) = 2x^3 + 8x^2 - 22x - 60$?

$$\begin{array}{r|rrrr} -2 & 2 & 8 & -22 & -60 \\ & & -4 & -8 & 60 \\ \hline & 2 & 4 & -30 & 0 \end{array} \qquad \begin{array}{r|rrrr} 6 & 2 & 8 & -22 & -60 \\ & & 12 & 120 & \\ \hline & 2 & 20 & 98 & \end{array}$$

$\boxed{(x+2) \text{ Factor}}$ $\boxed{(x-6) \text{ Not a factor}}$

Find **all** the zeros of the following functions

2. $g(x) = x^3 + 4x^2 + 4x$

$x(x^2+4x+4)$

$x(x+2)(x+2)$

$\boxed{\text{zeros: } 0, -2}$

3. $h(x) = 3x^3 - 2x^2 - 3x + 2$

$\frac{\pm 1, \pm 2}{\pm 1, \pm 3} = \pm 1, \pm \frac{1}{3}, \pm 2, \ldots$

$$\begin{array}{r|rrrr} -1 & 3 & -2 & -3 & 2 \\ & & -3 & 5 & -2 \\ \hline & 3 & -5 & 2 & 0 \end{array}$$

$3x^2 - 5x + 2$

$(x-1)(3x-2)$

$\boxed{\text{Zeros: } -1, 1, 2/3}$

4. $f(x) = x^4 + x^3 - 14x^2 - 2x + 24$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$$\begin{array}{r|rrrrr} -4 & 1 & 1 & -14 & -2 & 24 \\ & & -4 & 12 & 8 & -24 \\ \hline 3 & 1 & -3 & -2 & 6 & 0 \\ & & 3 & 0 & -6 & \\ \hline & 1 & 0 & -2 & 0 & \end{array}$$

$x^2 - 2$

$\frac{0 \pm \sqrt{0 - 4(1)(-2)}}{2} = \frac{\pm\sqrt{8}}{2} = \frac{\pm 2\sqrt{2}}{2} = \pm\sqrt{2}$

$\boxed{\text{zeros: } -4, 3, \pm\sqrt{2}}$

5. $k(x) = 7x^3 + x^2 - 28x - 4$

$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 7} = \pm 1, \pm \frac{1}{7}, \pm 2, \pm \frac{2}{7}, \pm 4, \ldots$

$$\begin{array}{r|rrrr} 2 & 7 & 1 & -28 & -4 \\ & & 14 & 30 & 4 \\ \hline & 7 & 15 & 2 & 0 \end{array}$$

$7x^2 + 15x + 2$

$(x+2)(7x+1)$

$\boxed{\text{zeros: } 2, -2, -\frac{1}{7}}$

Given the following zeros and multiplicities, write a function in factored form

6. 2 (multiplicity of 3), 5, –7(multiplicity of 2)

$(x-2)^3(x-5)(x+7)^2$

7. 4, 2(multiplicity of 5), -3

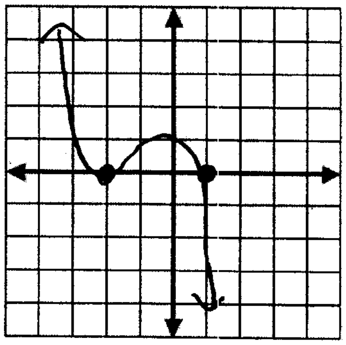
$(x-4)(x-2)^5(x+3)$

8. Given $g(x) = 3x^3 - 8x^2 + 3x + 2$, use the rational root theorem to determine which of the following are ***possible zeros*** of the function.

a. 2   b. -3   c. 4   d. $-\frac{2}{3}$ e. $\frac{3}{4}$

For the following functions, find the zeros, state the end behavior using limit notation, and graph the function.

9. $f(x) = -(x+2)^2(x-1)$



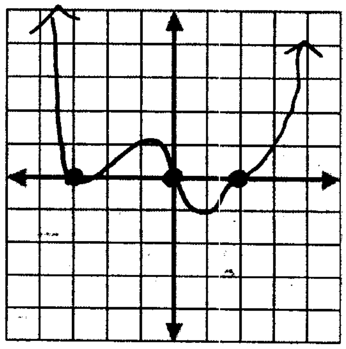
zeros: $x = -2, 1$

End Behavior:

$\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -\infty$

10. $h(x) = x(x+3)^2(x-2)^3$



zeros: $x = 0, -3, 2$

End Behavior:

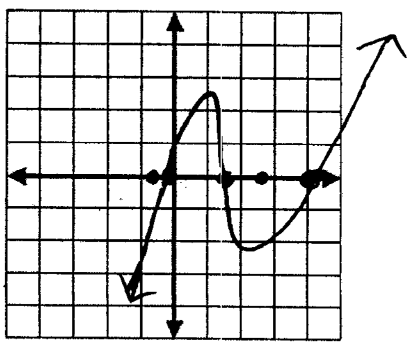
$\lim_{x \to -\infty} h(x) = \infty$

$\lim_{x \to \infty} h(x) = \infty$

11. $f(x) = x^3 - 10x^2 + 14x + 16$

$$\begin{array}{r|rrrr} 8 & 1 & -10 & 14 & 16 \\ & & 8 & -16 & -16 \\ \hline & 1 & -2 & -2 & \end{array}$$

$x^2 - 2x - 2$



zeros: $x = 8, 1 \pm \sqrt{3}$

End Behavior:

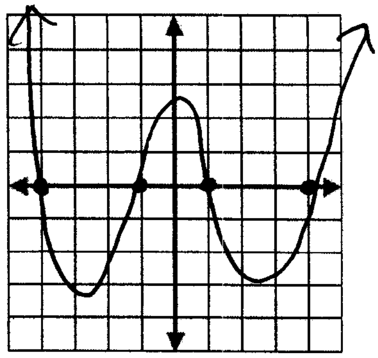
$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

$g(x) = x^4 - 17x^2 + 16$

12. ~~$g(x) = x^4 + 15x^2 - 16$~~

$g(x) = (x+1)(x-1)(x+4)(x-4)$



zeros: $x = -4, -1, 1, 4$

End Behavior: $\lim\limits_{x \to -\infty} g(x) = \infty$

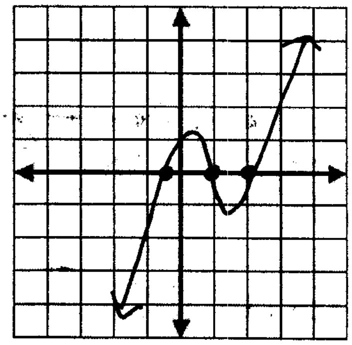
$\lim\limits_{x \to \infty} g(x) = \infty$

13. $3x^3 - 8x^2 + 3x + 2 = f(x)$

$$\begin{array}{r|rrrr} 1 & 3 & -8 & 3 & 2 \\ & & 3 & -5 & -2 \\ \hline & 3 & -5 & -2 & ☺ \end{array}$$

$3x^2 - 5x - 2$

$f(x) = (x-1)(x-2)(3x+1)$



zeros: $x = 1, 2, -1/3$

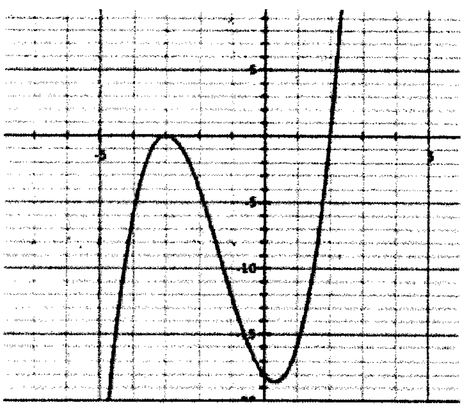
End Behavior:

$\lim\limits_{x \to -\infty} f(x) = -\infty$

$\lim\limits_{x \to \infty} f(x) = \infty$

Given the following graphs analyze the functions

14.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, -3) \cup (0.5, \infty)$

Decreasing: $(-3, 0.5)$

#/type max: 1 local

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x-intercept(s): $(-3, 0)$ $(2, 0)$

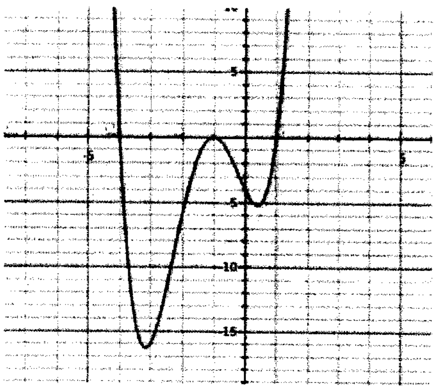
y-intercept: $(0, -8)$

End Behavior:

$\lim\limits_{x \to -\infty} f(x) = -\infty$

$\lim\limits_{x \to \infty} f(x) = \infty$

15.



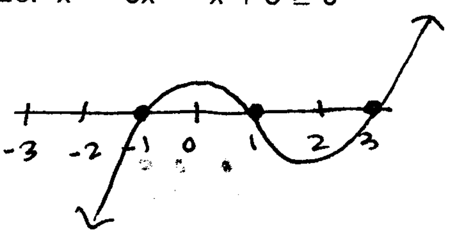
Domain: $(-\infty, \infty)$
Range: $[-16, \infty)$
Increasing: $(-3.2, -1) \cup (0.5, \infty)$
Decreasing: $(-\infty, -3.2) \cup (-1, 0.5)$
#/type max: 1 local
#/type min: 1 local, 1 global
x-intercept(s): $(-4,0)$ $(-1,0)$, $(1,0)$
y-intercept: $(0,-4)$
End Behavior:

$$\lim_{x\to-\infty} f(x) = \infty$$

$$\lim_{x\to\infty} f(x) = \infty$$

Solve the following polynomial inequalities

16. $x^3 - 3x^2 - x + 3 \geq 0$



$\boxed{[-1, 1] \cup [3, \infty)}$

17. $x^3 - 7x^2 + 10x + 6 < 0$

Zeros:

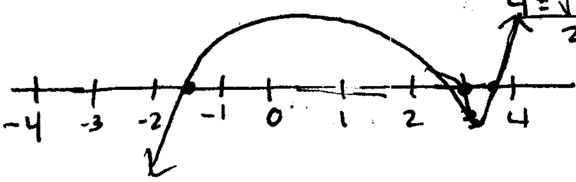
$$\begin{array}{r|rrrr} 3 & 1 & -7 & 10 & 6 \\ & & 3 & -12 & -6 \\ \hline & 1 & -4 & -2 & 0 \end{array}$$

$x^2 - 4x - 2$

$\frac{4 \pm \sqrt{4^2 - 4(1)(-2)}}{2}$

$\frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2}$

$2 \pm \sqrt{6}$



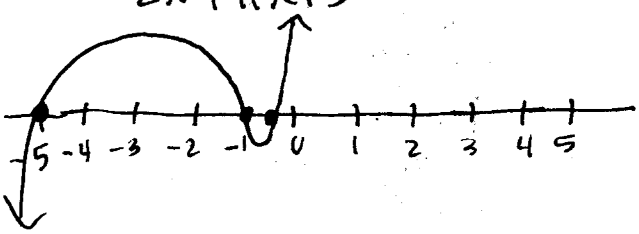
$\boxed{(-\infty, 2-\sqrt{6}) \cup (3, 2+\sqrt{6})}$

18. ~~$2x^3 - 7x^2 + x + 5 > 0$~~

$2x^3 + 13x^2 + 16x + 5 > 0$

$$\begin{array}{r|rrrr} -1 & 2 & 13 & 16 & 5 \\ & & -2 & -11 & -5 \\ \hline & 2 & 11 & 5 & 0 \end{array}$$

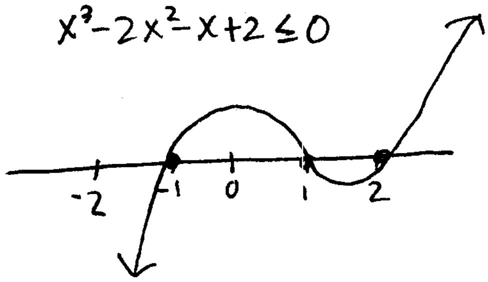
$2x^2 + 11x + 5$



$\boxed{(-5, -1) \cup (-\frac{1}{2}, \infty)}$

19. ~~$x^3 + x^2 - x + 2 \leq 0$~~

$x^3 - 2x^2 - x + 2 \leq 0$



$\boxed{(-\infty, -1] \cup [1, 2]}$