

Perform the indicated operation and write the result in standard form.

$$1. (1+i)^2$$

$$(1+i)(1+i)$$

$$= 1+i+i+i^2$$

$$= 1+2i-1$$

$$= \boxed{2i}$$

$$2. \sqrt{-16}$$

$$= \boxed{4i}$$

$$3. (3-2i)+(-2+5i)$$

$$= \boxed{1+3i}$$

$$4. (5-7i)-(3-2i)$$

$$= \boxed{2-5i}$$

$$5. (1+2i)(3-2i)$$

$$= 3-4i^2$$

$$= 3+4$$

$$= \boxed{7}$$

$$6. \sqrt{-75}$$

$$= \boxed{5i\sqrt{3}}$$

Solve the following polynomials

$$7. x^2 - 6x + 13 = 0$$

$$\frac{6 \pm \sqrt{36 - 4(13)}}{2} = \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6 \pm 4i}{2} = \boxed{3 \pm 2i}$$

$$8. x^2 + 24 = 0$$

$$\sqrt{x^2} = \sqrt{-24}$$

$$x = \boxed{2i\sqrt{6}}$$

State how many complex and real zeros the function has

$$9. f(x) = x^2 - 2x + 7$$

2 complex
- 2 complex imaginary

$$10. f(x) = x^3 - x + 3$$

3 complex
- 1 real
- 2 complex imaginary

$$11. f(x) = x^4 - 2x^2 + 3x - 4$$

4 complex
- 2 real
- 2 complex imaginary

$$12. f(x) = x^5 - 2x^2 - 3x + 6$$

5 complex
- 1 real
- 4 complex imaginary

Write a polynomial in **factored form** with minimum degree given the following zeros.

13. 1, 3i

$$f(x) = (x-1)(x+3i)(x-3i)$$

14. 3, 3+2i

$$f(x) = (x-3)(x-(3+2i))(x-(3-2i))$$

$$= (x-3)(x-3-2i)(x-3+2i)$$

Write a polynomial in **standard form** with minimum degree given the following zeros.

15. 1+2i, 1-2i

$$f(x) = (x-(1+2i))(x-(1-2i))$$

$$= (x-1-2i)(x-1+2i)$$

$$= x^2 - x + 2xi - x + 1 - 2i - 2xi + 2i - 4i^2$$

$$= \boxed{x^2 - 2x + 5}$$

16. -2, 1+2i

$$f(x) = (x+2)(x-(1+2i))$$

$$= (x+2)(x-1-2i)$$

$$= x^2 - x - 2xi + 2x - 2 - 4i$$

$$= \boxed{x^2 - 2xi + x - 4i - 2}$$

Find all the zeros and write a linear factorization of the function.

17. $f(x) = x^3 + 4x - 5$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 4 & -5 \\ & & 1 & 1 & \\ \hline & 1 & 1 & 5 & \end{array}$$

$$x^2 + x + 5 = 0$$

$$\frac{-1 \pm \sqrt{1-4(5)}}{2} = \frac{-1 \pm \sqrt{-19}}{2}$$

Zeros: $x = 1, -\frac{1}{2} \pm \frac{\sqrt{19}}{2}i$

$$f(x) = (x-1)(x - (-\frac{1}{2} \pm \frac{\sqrt{19}}{2}i))$$

19. $f(x) = x^4 + x^3 + 5x^2 - x - 6$

$$\begin{array}{r|rrrrrr} -1 & 1 & 1 & 5 & -1 & -6 \\ & & -1 & 0 & -5 & \\ \hline 1 & 1 & 0 & 5 & -6 & \\ & & 1 & 1 & \\ \hline & 1 & 1 & 6 & \end{array}$$

$$x^2 + x + 6 = 0$$

$$\frac{-1 \pm \sqrt{1-4(6)}}{2} = \frac{-1 \pm \sqrt{-23}}{2}$$

Zeros: $x = -1, 1, -\frac{1}{2} \pm \frac{\sqrt{23}}{2}i$

$$f(x) = (x+1)(x-1)(x - (-\frac{1}{2} \pm \frac{\sqrt{23}}{2}i))$$

18. $f(x) = x^3 - 10x^2 + 44x - 69$

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 44 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & \end{array}$$

$$x^2 - 7x + 23 = 0$$

$$\frac{7 \pm \sqrt{7^2 - 4(23)}}{2} = \frac{7 \pm \sqrt{49 - 92}}{2} = \frac{7 \pm \sqrt{-43}}{2}$$

Zeros: $x = 3, \frac{7}{2} \pm \frac{\sqrt{43}}{2}i$

$$f(x) = (x-3)(x - (\frac{7}{2} \pm \frac{\sqrt{43}}{2}i))$$

20. $f(x) = 3x^4 + 8x^3 + 6x^2 + 3x - 2$

$$\begin{array}{r|rrrrrr} -2 & 3 & 8 & 6 & 3 & -2 \\ & & -6 & -4 & -4 & \\ \hline \frac{1}{3} & 3 & 2 & 2 & -1 & \\ & & 1 & 1 & \\ \hline & 3 & 3 & 3 & \end{array}$$

$$3x^2 + 3x + 3 = 0$$

$$\frac{-3 \pm \sqrt{3^2 - 4(3)(3)}}{2} = \frac{-3 \pm \sqrt{9 - 36}}{2} = \frac{-3 \pm 3i\sqrt{3}}{2}$$

Zeros: $x = -2, \frac{1}{3}, -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$

$$f(x) = (x+2)(x-\frac{1}{3})(x - (-\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i))$$

long division

Use the given zero to find all the zeros of the function.

21. $1+i$, $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$

$1-i$
 $(x - (1+i))(x - (1-i))$
 $(x - 1 - i)(x - 1 + i)$
 $(x^2 - 2x + 1 + 1)$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$
 22. $4i$, $f(x) = x^4 + 13x^2 - 48$

$-4i$
 $(x + 4i)(x - 4i)$
 $(x^2 + 16)(x^2 - 3)$

$x^2 - 3 = 0$

$x^2 = 3$
 $x = \pm\sqrt{3}$

$\{4i, -4i, \sqrt{3}, -\sqrt{3}\}$

$$\begin{array}{r} x^2 - 3 \\ x^2 - 3 \overline{) x^4 - 2x^3 - x^2 + 6x - 6} \\ \underline{-x^4 + 2x^2 - 2x^2} \\ -3x^2 + 6x - 6 \\ \underline{+3x^2 - 6x + 6} \\ 0 \end{array}$$

$\{1+i, 1-i, \sqrt{3}, -\sqrt{3}\}$

$$\begin{array}{r} x^2 - 3 \\ x^2 + 16 \overline{) x^4 + 0x^3 + 13x^2 + 0x - 48} \\ \underline{-x^4} \\ -16x^2 \\ \underline{-3x^2} \\ -48 \\ \underline{+3x^2} \\ +48 \\ 0 \end{array}$$

23. $3-2i$, $f(x) = x^4 - 6x^3 + 11x^2 + 12x - 26$

$3+2i$
 $(x - (3-2i))(x - (3+2i))$
 $(x - 3 + 2i)(x - 3 - 2i)$
 $(x^2 - 6x + 9 + 4)$
 $(x^2 - 6x + 13)(x^2 - 2)$

$x^2 - 2 = 0$
 $x^2 = 2$
 $x = \pm\sqrt{2}$

$$\begin{array}{r} x^2 - 2 \\ x^2 - 6x + 13 \overline{) x^4 - 6x^3 + 11x^2 + 12x - 26} \\ \underline{-x^4 + 6x^3 - 13x^2} \\ -2x^2 + 12x - 26 \\ \underline{-2x^2 + 12x - 26} \\ 0 \end{array}$$

$\{3-2i, 3+2i, \sqrt{2}, -\sqrt{2}\}$

Expand the following

24. $(5x+y)^4 = (5x)^4(y)^0 + 4(5x)^3(y)^1 + 6(5x)^2(y)^2 + 4(5x)^1(y)^3 + (5x)^0(y)^4$

$625x^4 + 500x^3y + 150x^2y^2 + 20xy^3 + y^4$

25. $(2+3y)^5 =$

$(2)^5(3y)^0 + 5(2)^4(3y)^1 + 10(2)^3(3y)^2 + 10(2)^2(3y)^3 + 5(2)^1(3y)^4 + (2)^0(3y)^5$

$32 + 240y + 720y^2 + 1080y^3 + 810y^4 + 243y^5$

$243y^5 + 810y^4 + 1080y^3 + 720y^2 + 240y + 32$

Synthetic division

Use the given zero to find all the zeros of the function.

21. $1+i$, $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$

$$\begin{array}{r|rrrrr} 1+i & 1 & -2 & -1 & 6 & -6 \\ & & 1+i & -2 & -3-3i & 6 \\ \hline 1-i & 1 & -1+i & -3 & 3-3i & 0 \\ & & \cancel{1-i} & 0 & -3+3i & \\ \hline & 1 & -3 & 0 & 0 & \\ & & x^2-3=0 & & & \\ & & x = \pm\sqrt{3} & & & \end{array}$$

Zeros: $x = 1+i, 1-i, \pm\sqrt{3}$

$$\begin{aligned} (1+i)(-1+i) &= -1 + i - i + i^2 \\ &= -1 - 1 \\ &= -2 \\ (1+i)(3-3i) &= 3 - 3i + 3i - 3i^2 \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

22. $4i$, $f(x) = x^4 + 13x^2 - 48$

$$\begin{array}{r|rrrrr} 4i & 1 & 0 & 13 & 0 & -48 \\ & & 4i & -10 & -12i & 48 \\ \hline -4i & 1 & 4i & -3 & -12i & 0 \\ & & -4i & 0 & 12i & \\ \hline & 1 & 0 & -3 & 0 & \\ & & x^2-3=0 & & & \\ & & x = \pm\sqrt{3} & & & \end{array}$$

Zeros: $x = 4i, -4i, \sqrt{3}, -\sqrt{3}$

23. $3-2i$, $f(x) = x^4 - 6x^3 + 11x^2 + 12x - 26$

$$\begin{array}{r|rrrrr} 3-2i & 1 & -6 & 11 & 12 & -26 \\ & & 3-2i & -13 & -6+4i & 26 \\ \hline 3+2i & 1 & -3-2i & -2 & 6+4i & 0 \\ & & 3+2i & 0 & -6-4i & \\ \hline & 1 & 0 & -2 & 0 & \\ & & x^2-2=0 & & & \\ & & x = \pm\sqrt{2} & & & \end{array}$$

Zeros: $x = 3-2i, 3+2i, \sqrt{2}, -\sqrt{2}$

$$\begin{aligned} (3-2i)(-3-2i) &= -9 - 6i + 6i + 4i^2 \\ &= -9 - 4 = -13 \\ (3-2i)(6+4i) &= 18 + 12i - 12i - 8i^2 \\ &= 18 + 8 = 26 \end{aligned}$$

Expand the following

24. $(5x+y)^4$

$$625x^4 + 500x^3y + 150x^2y^2 + 20xy^3 + y^4$$

25. $(2+3y)^5$

$$243y^5 + 810y^4 + 1080y^3 + 720y^2 + 240y + 32$$