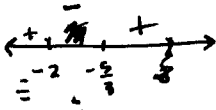Name: **KEY**

Find the following information and graph each rational function:

✶ 1. $f(x) = \dfrac{3x+5}{x+2}$

Domain: $(-\infty, -2) \cup (-2, \infty)$
Range:
x- int: $(-\frac{5}{3}, 0)$
Vertical Asymptote: $x = -2$
Horizontal Asymptote: $y = 3$
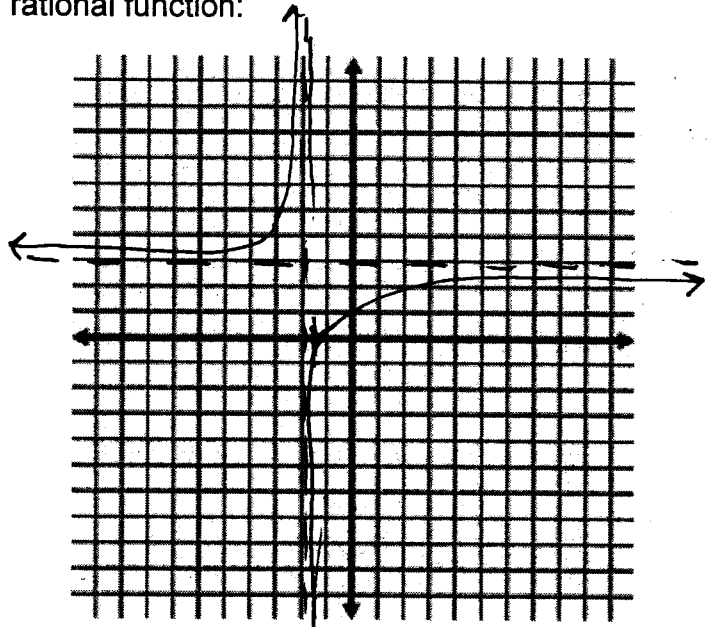Increasing: $(-\infty, -2) \cup (-2, \infty)$
Decreasing: never
End Behavior:
$\lim\limits_{x \to -\infty} f(x) = 3$ $\lim\limits_{x \to \infty} f(x) = 3$
Asymptote Behavior:
$\lim\limits_{x \to -2^-} f(x) = \infty$ $\lim\limits_{x \to -2^+} f(x) = -\infty$

✶ 2. $f(x) = \dfrac{x^2 - 4x - 5}{x+3}$ $(x-5)(x+1)$

Domain: $(-\infty, -3) \cup (-3, \infty)$
Range: $(-\infty, -15] \cup [-2, \infty)$
x- int: $(5, 0)(-1, 0)$
Vertical Asymptote: $x = -3$
Horizontal Asymptote: $y = x$
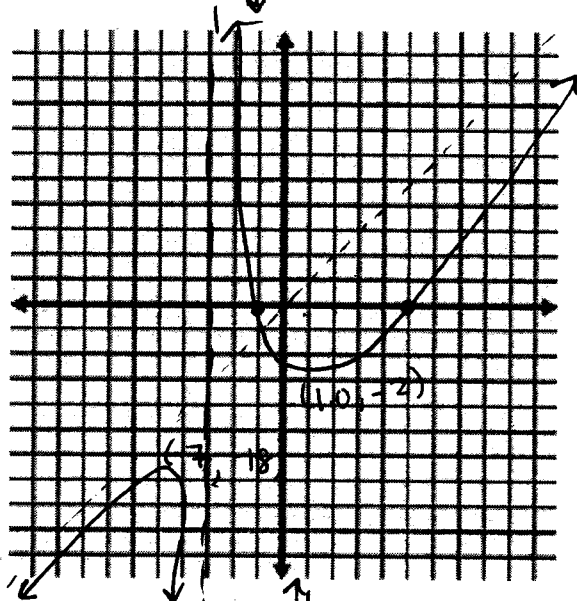Increasing: $(-\infty, -7) \cup (1, \infty)$
Decreasing: $(-7, -3) \cup (-3, 1)$
End Behavior:
$\lim\limits_{x \to -\infty} f(x) = -\infty$ $\lim\limits_{x \to \infty} f(x) = \infty$
Asymptote Behavior:
$\lim\limits_{x \to -3^-} f(x) = -\infty$ $\lim\limits_{x \to -3^+} f(x) = \infty$

3. $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 1}$ $\dfrac{(x-3)(x+1)}{(x-1)(x+1)}$

Domain: $(-\infty, 1) \cup (1, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$
x- int: $(3, 0)$
Vertical Asymptote: $x = 1$
Horizontal Asymptote: $y = 1$
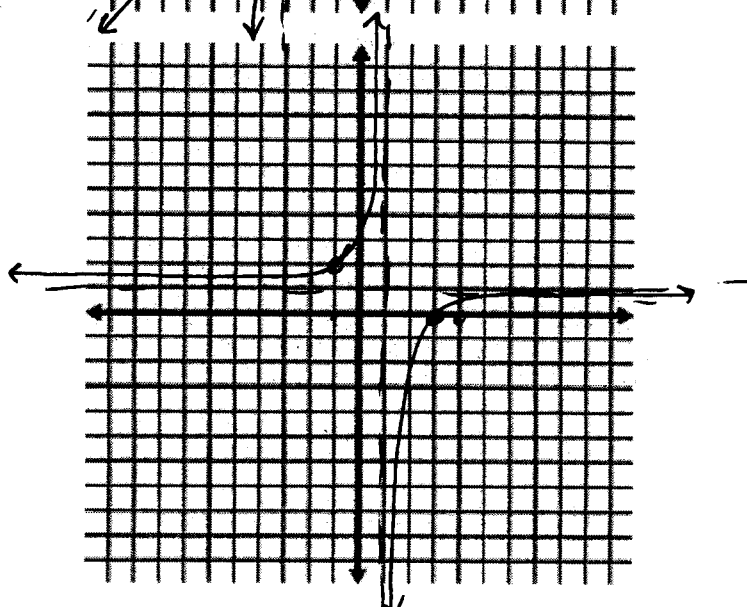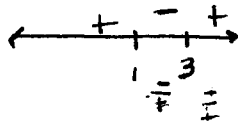Increasing: $(-\infty, 1) \cup (1, \infty)$
Decreasing: Never
End Behavior:
$\lim\limits_{x \to -\infty} f(x) = 1$ $\lim\limits_{x \to \infty} f(x) = 1$
Asymptote Behavior:
$\lim\limits_{x \to 1^-} f(x) = \infty$ $\lim\limits_{x \to 1^+} f(x) = -\infty$

Solve the following inequalities using a sign chart:

4. $\dfrac{x+5}{x^2-4} < 0$
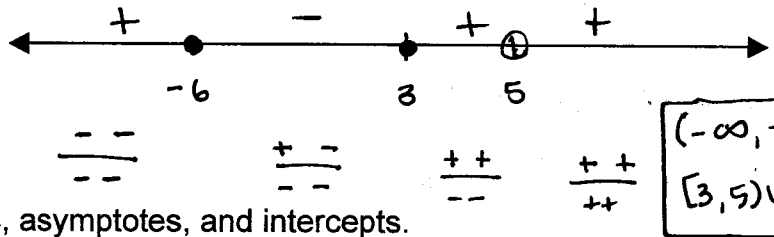
$\dfrac{x+5}{(x+2)(x-2)} < 0$

VA: $x = -2$
$x = 2$

X-int: $x = -5$

$\dfrac{-}{-\ -}$ $\dfrac{+}{-\ -}$ $\dfrac{+}{+\ -}$ $\dfrac{+}{+\ +}$

$\boxed{(-\infty, -5) \cup (-2, 2)}$
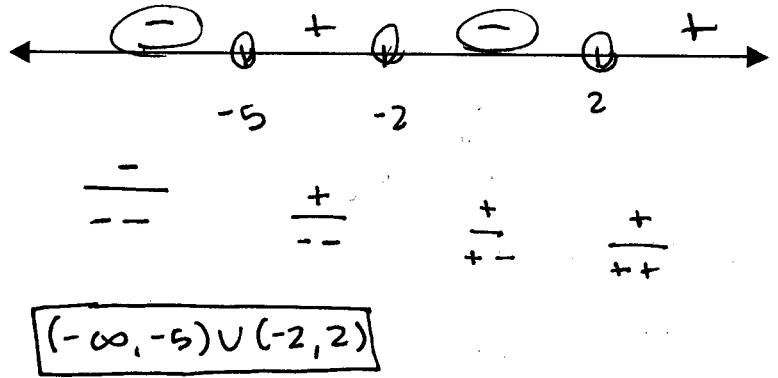
5. $\dfrac{x^2+3x-18}{x^2-10x+25} \geq 0$

$\dfrac{(x+6)(x-3)}{(x-5)(x-5)} \geq 0$ VA: $x = 5$
X-int: $x = -6$
$x = 3$

$\dfrac{-\ -}{-\ -}$ $\dfrac{+\ -}{-\ -}$ $\dfrac{+\ +}{-\ -}$ $\dfrac{+\ +}{+\ +}$

$\boxed{(-\infty, -6] \cup [3, 5) \cup (5, \infty)}$

Given the following functions, find all holes, asymptotes, and intercepts.

6. $f(x) = \dfrac{x-3}{x^2+6x+5}$

$f(x) = \dfrac{x-3}{(x+5)(x+1)}$ VA: $x = -5$ X-int: $(3, 0)$
$\qquad\qquad\qquad\qquad x = -1$ y-int: $\left(0, -\dfrac{3}{5}\right)$
$\qquad\qquad\qquad$ HA: $y = 0$

7. $f(x) = -\dfrac{(x^2-4)}{(x+1)}$

$f(x) = -\dfrac{(x+2)(x-2)}{x+1}$ VA: $x = -1$ X-int: $(-2, 0)\ (2, 0)$
$\qquad\qquad\qquad\qquad$ OA: $y = x$ y-int: $(0, 4)$

8. $f(x) = \dfrac{x(x+4)^2(x-5)}{(x-5)^3(x+1)^2}$

$f(x) = \dfrac{x(x+4)^2}{(x-5)(x+1)^2}$ Hole @ $x = 5$
$\qquad\qquad\qquad\qquad$ VA: $x = 5$ X-int: $(0,0), (-4, 0)$
$\qquad\qquad\qquad\qquad\quad x = -1$ y-int: $(0, 0)$
$\qquad\qquad\qquad\qquad$ HA: $y = 1$

(Transformations)

Describe how the graph of g(x) is related to the graph $f(x) = \frac{1}{x}$.

9. $g(x) = \frac{5}{x} - 3$

Shifted down 3

10. $g(x) = \frac{-1}{x} + 5$

up 5

Flipped over x-axis

11. $g(x) = -\frac{1}{(x-2)} + 4$

Right 2
up 4
Flipped over x-axis

12. How do you find the asymptotes (vertical and end behavior) of a rational function?

Vertical : x-values that make the denominator 0.

end behavior :

13. Mary and some of her friends are thinking about renting a car while staying at a beach resort for a vacation. The cost per person for staying at the beach resort is $300, and the cost of the car rental is $220. If the friends agree to share the cost of the car rental, what is the minimum number of people who must go on the trip so that the total cost for each person is no more than $350?
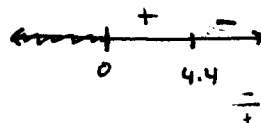
$300 + \frac{220}{x} \leq 350$

$-350 \qquad -350$

$\frac{220}{x} - 50 \leq 0$

$\frac{220}{x} - 50 \cdot \frac{x}{x} \leq 0$

$\frac{220 - 50x}{x} \leq 0$

VA: x = 0
x-int: K = 4.4

> 4.4

[number line: + between 0 and 4.4, − after 4.4]
0   4.4

$\frac{-}{+}$

Minimum of 5
People

14. A basketball team has won 16 games out of 23 games played, for a winning percentage (expressed as a decimal) of $\frac{16}{23} \approx 0.696$. How many consecutive games must the team win to raise its winning percentage to 0.750?

$\frac{16+x}{23+x} \geq 0.750$

$\frac{16+x}{23+x} - 0.750 = 0$

$\frac{16+x}{23+x} - \frac{17.25 + 0.75x}{23+x} = 0$

$\frac{.25x - 1.25}{23 + x} = 0$

$.25x - 1.25 = 0$
$+1.25 \quad +1.25$

$\frac{.25x}{.25} = \frac{1.25}{.25}$

x = 5 more games