20pts

Review Unit 7
Secondary III

Name: KEY
Date: ______________ Class: ________

Write an explicit and recursive rule for the following

1. 9, 27, 81, 243,...

Explicit: $f(n) = 9 \cdot 3^n,\ n \geq 0$

Recursive: $f(n) = f(n-1) \cdot 3,\ f(0) = 9$
$n \geq 1$

2. 4, −3, −10, −17, ...

Explicit: $f(n) = 4 - 7n,\ n \geq 0$

Recursive: $f(0) = 4,\ f(n) = f(n-1) - 7,\ n \geq 1$

3. Find the 12th term of the geometric sequence 5, 15, 45,...

$f(n) = 5 \cdot 3^n$

$f(11) = 5 \cdot 3^{11} = \boxed{885,735}$

4. If the first three terms of a geometric sequence are 3, 12, and 48, what is the seventh term?

$f(n) = 3 \cdot 4^n$

$f(6) = 3 \cdot 4^6 = \boxed{12,288}$

Find the stated term for the following sequences

5. −3, −6, −12, −24, ...; 9th term

$f(n) = -3 \cdot 2^n$

$f(8) = -3 \cdot 2^8$

6. 4, −12, 36, −108, ...; 11th term

$\sum_{k=0}^{n} a_0 (r)^k = \frac{a_0(1-r^{n+1})}{1-r}$

Find the sum of the geometric series.

7. 4 + 16 + 64 + 256 + ... + 16,384

$16,384 = 4 \cdot 4^k$
$4096 = 4^k$
$k = 6$

$\sum_{k=0}^{6} 4 \cdot 4^k = \frac{4(1-4^7)}{1-4}$

$= \boxed{21844}$

8. 3 − 6 + 12 − 24 + ... − 1536

$-1536 = 3 \cdot -2^n$
$-512 = -2^n$
$n = 9$

$\sum_{k=0}^{9} 3 \cdot -2^k = \frac{3(1-(-2)^{10})}{1--2}$

$= \boxed{-1023}$

9. −2 − 6 − 18 − 54 − 162

$\sum_{k=0}^{4} -2 \cdot 3^k = \frac{-2(1-3^5)}{1-3}$

$= \boxed{-242}$

10. −2 + 8 − 32 + ... +2048

$2048 = -2 \cdot -4^n$
$-1024 = -4^n$
$n = 5$

$\sum_{k=0}^{5} -2 \cdot -4^k = \frac{-2(1-(-4)^6)}{1--4}$

$= \boxed{1638}$

Evaluate the following

11. $\sum_{n=1}^{5} 2n+1$

$= 2(1)+1 + 2(2)+1 + 2(3)+1$
$+ 2(4)+1 + 2(5)+1$

$= 3+5+7+9+11$

$= \boxed{35}$

12. $\sum_{k=1}^{3} k^2 - 1$

$= 1^2-1 + 2^2-1 + 3^2-1$

$= 0+3+8$

$= \boxed{11}$

13. A geometric sequence that has an initial value 2, ends with -4374 and has a common ratio of -3, how many terms are in the sequence?

$-4374 = 2 \cdot -3^n$

$-2187 = -3^n$ $\quad \boxed{8 \text{ terms}}$

$n = 7$

14. A geometric sequence that begins with 2000 and successively decreases by 10%, find the 8th term.

$f(n) = 2000(1-.10)^n = 2000(0.9)^n$

$f(7) = 2000(0.9)^7 = \boxed{956.59}$

Find the domain and range for the following functions

15. $f(x) = 3^{x-2} - 1$

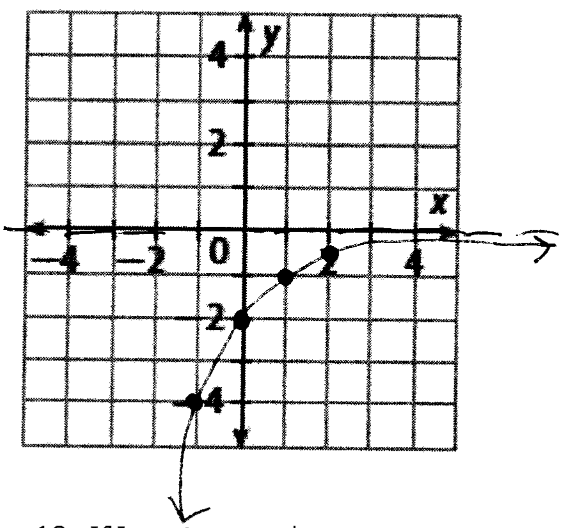
Domain: $(-\infty, \infty)$
Range: $(-1, \infty)$

16. $f(x) = \left(\frac{1}{3}\right)^x + 2$

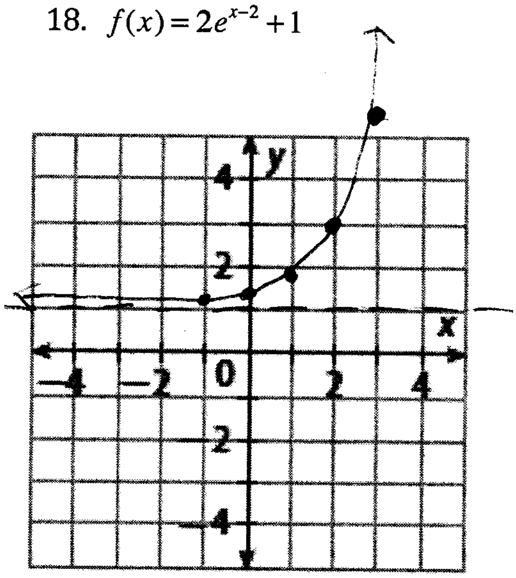
Domain: $(-\infty, \infty)$
Range: $(2, \infty)$

Graph the following and label any asymptotes or intercepts

17. $g(x) = -2\left(\frac{1}{2}\right)^x$



18. $f(x) = 2e^{x-2} + 1$



19. If Jane invests $4,200 at an 8% interest **compounded continuously**, how much money will there be after 10 years?

Answer the following questions with the following: an investment of $2000 that earns 3.4% interest

$A(t) = 4200e^{.08(10)}$

$= \boxed{\cancel{\$49,498.10}}$ $\boxed{\$9347.27}$

$2000, 3.4%

20. Write an equation to describe the value V(t) of the investment at time t if the interest is compounded monthly.

$$V(t) = 2000\left(1+\frac{.034}{12}\right)^{12t}$$

21. What is the value of the investment after 10 years if compounded annually?

$$V(10) = 2000\left(1+\frac{.034}{1}\right)^{1(10)} = \boxed{\$2794.06}$$

22. How long would it take for the investment to reach $10,000 if the interest is compounded annually?

$$10{,}000 = 2000(1+.034)^t$$
$$t \approx 48$$

$\boxed{\text{About 48 yrs}}$

23. A melting snowman is losing one-half of his weight each day. He originally weighed 128 pounds. Assuming that the outside temperature stays the same, how much does the snowman weigh after 5 days?

$$f(5) = 128(1/2)^5$$
$$= 4$$

$\boxed{\text{4 lbs}}$

24. A car with a cost of $25,000 is decreasing in value at a rate of 10% each year. The function $g(t) = 25{,}000(0.9)^t$ gives the value of the car after t years. When will the value of the car be about $12,000?

$$12{,}000 = 25{,}000(0.9)^t$$
$$t \approx 6.966$$

$\boxed{\text{About 7 years}}$

25. The population of a town was estimated to be about 5000 in 1980. The exponential growth function that models this situation is $P(t) = 5000e^{0.044t}$, where t is the time in years after 1980, and $P(t)$ is the population at time t.

a. What is the initial amount?

5000

b. What is the population after 20 years?

About 12,054 people