10-1 Solving Triangles

Objectives:

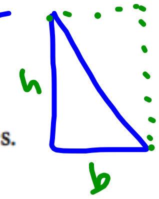
I can calculate the area of a non-right triangle

I can use inverse trig functions

I can solve a right triangle for lengths and sides

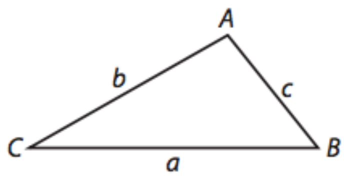
Explore — Deriving an Area Formula

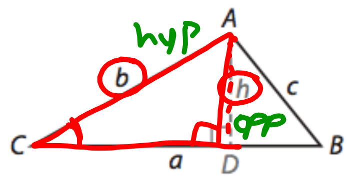
$A = \frac{1}{2} b \cdot h$



You can use trigonometry to find the area of a triangle without knowing its height.

(A) Suppose you draw an altitude $\overline{AD}$ to side $\overline{BC}$ of $\triangle ABC$. Then write an equation using a trigonometric ratio in terms of $\angle C$, the height h of $\triangle ABC$, and the length of one of its sides.





$b \cdot \sin C = \frac{h}{b} \cdot b$

(B) Solve your equation from Step A for h.

$h = b \cdot \sin C$

(C) Complete this formula for the area of $\triangle ABC$ in terms of h and another of its side lengths: Area $= \frac{1}{2}$ base · height

(D) Substitute your expression for h from Step B into your formula from Step C.

$A = \frac{1}{2} a \cdot b \cdot \sin C$

2. Suppose you used a trigonometric ratio in terms of $\angle B$, h, and a different side length. How would this change your findings? What does this tell you about the choice of sides and included angle?

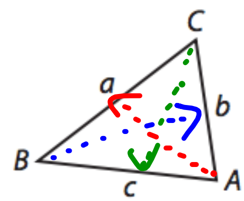
side lengths

angle

Area formulas of a non-right triangles

Area Formula for a Triangle in Terms of its Side Lengths

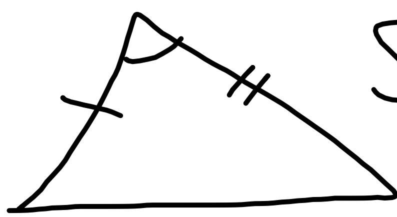
The area of $\triangle ABC$ with sides a, b, and c can be found using the lengths of two of its sides and the sine of the included angle: Area $= \frac{1}{2}bc \sin A$, Area $= \frac{1}{2}ac \sin B$, or Area $= \frac{1}{2} ab \sin C$.



lower case
side

upper case
angles

Area:



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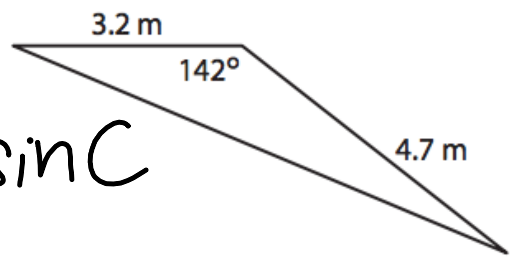
Example 1 Find the area of each triangle to the nearest tenth.

(A)



Area: $\frac{1}{2} a \cdot b \cdot \sin C$

$= \frac{1}{2}(3.2)(4.7)\sin 142^\circ$ → calc deg

$\boxed{4.6 \text{ m}^2}$

(B) In $\triangle DEF$, $DE = 9$ in., $DF = 13$ in., and $m\angle D = 57°$.

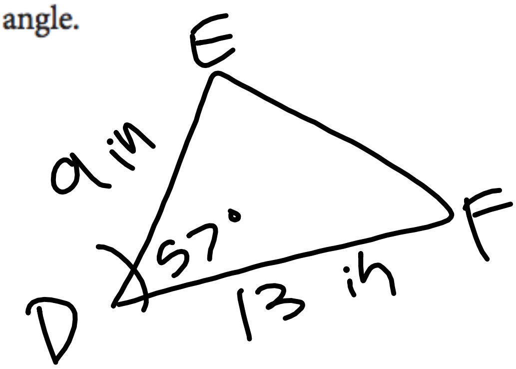
Sketch $\triangle DEF$ and check that $\angle D$ is the included angle.

Find the area

$$A = \frac{1}{2} a \cdot b \cdot \sin C$$

$$= \frac{1}{2}(9)(13)\sin 57°$$

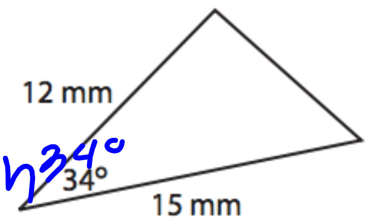
$$= \boxed{49.1 \text{ in}^2}$$



Your Turn

Find the area of each triangle to the nearest tenth.

3.



$$\text{Area} = \frac{1}{2}(12)(15)\sin 34^\circ$$

$$= \boxed{50.3\ mm^2}$$

To "solve" a triangle means to find ALL side lengths and angle measures.

a =
b =
c =

A =
B =
C =

REMEMBER

-Triangles have an angle sum of 180 degrees → 2 angles

-Pythagorean Theorem to find a missing side when you know two → 2 sides

-Inverse Trig to find a missing angle

$\sqrt{x^2}$ $\quad$ $+$ / $-$ $\quad$ $\cdot$ / $\div$ $\quad$ $\cos^{-1}\cos A$

Once you know the sine, cosine or the tangent of an acute angle, then you can use a calculator to find the measure of the angle.

For acute angle A:

If $\sin A = x$, then $\sin^{-1}(x) = m\measuredangle A$

angle trig ratio (sides) sides angle

If $\cos A = x$, then $\cos^{-1}(x) = m\measuredangle A$

If $\tan A = x$, then $\tan^{-1}(x) = m\measuredangle A$

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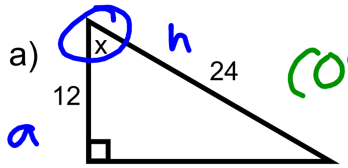
Inverse Trig

Find the measure of the indicated angle to the nearest **degree** (hint: calculator mode)

a)

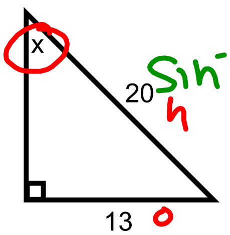


$\cos^{-1} \cos X = \cos^{-1} \frac{12}{24}$

$X = \cos^{-1}\left(\frac{12}{24}\right) \rightarrow$ calc

$\boxed{60^\circ}$

b)

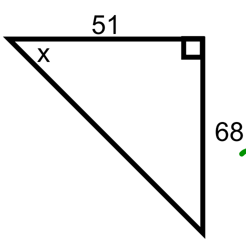


$\sin^{-1} \sin X = \sin^{-1} \frac{13}{20}$

$X = \sin^{-1}\left(\frac{13}{20}\right)$

$40.5 \rightarrow \boxed{41^\circ}$

c)



$\tan^{-1} \tan X = \tan^{-1} \frac{68}{51}$

$\boxed{53^\circ}$

Find the exact value. ~~Find ***ALL*** possible solutions.~~ acute angle $[0, \pi/2]$

$$\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$$

angle

$$\sin\frac{\pi}{6} = \boxed{\frac{1}{2}}$$

sides

$$\cos\left(\sec^{-1} 2\right)$$

$$\cancel{\cos\left(\cos^{-1}\frac{1}{2}\right)}$$

$$\cos\frac{\pi}{3} \to \frac{1}{2}$$

$$\sin^{-1}\left(\cos\frac{\pi}{3}\right)$$

$$\sin^{-1}\left(\frac{1}{2}\right) \to \boxed{\frac{\pi}{6}}$$

angle

$$\cos^{-1}\left(\sin\frac{3\pi}{2}\right)$$

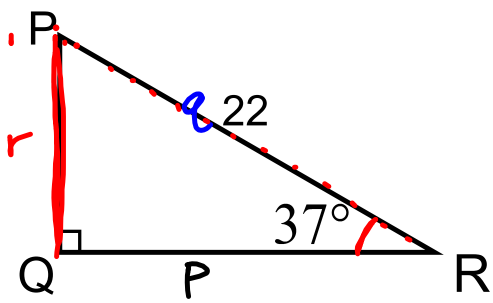
$0 - \pi$

$(0, -1)$

$$\cos^{-1}(-1)$$

$$\boxed{\pi}$$

Solve each right triangle. Round lengths to the nearest tenth and angles to the nearest degree.



$\measuredangle P = 53^\circ$ $\quad p = 17.6$

$\measuredangle Q = 90^\circ$ $\quad q = 22$

$\measuredangle R = 37^\circ$ $\quad r = 13.2$

$<P \rightarrow 180 - 90 - 37 =$

$22 \cdot \sin 37^\circ = \frac{r}{\cancel{22}} \cdot \cancel{22}$

$a^2 + b^2 = c^2 \rightarrow \text{hyp}$

$a^2 + 13.2^2 = 22^2$

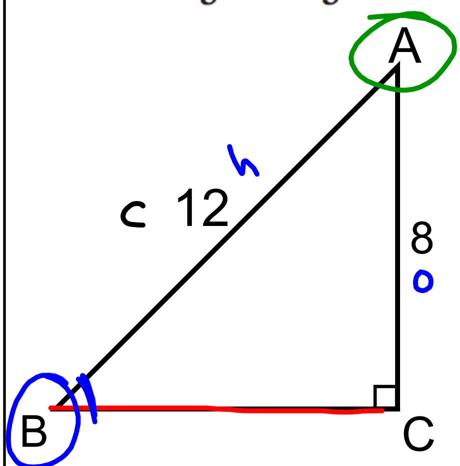
$a^2 + 174.24 = 484$

$\quad -174.24 \quad -174.24$

$\sqrt{a^2} = \sqrt{309.76}$

$a = 17.6$

Solve each right triangle. Round lengths to the nearest tenth and angles to the nearest degree.



$\measuredangle A = 48^\circ$ $\quad a = 8.9$

$\measuredangle B = 42^\circ$ $\quad b = 8$

$\measuredangle C = 90^\circ$ $\quad c = 12$

$a^2 + 8^2 = 12^2$

$a^2 + 64 = 144$

$\quad -64 \quad -64$

$\sqrt{a^2} = \sqrt{80}$

$a =$

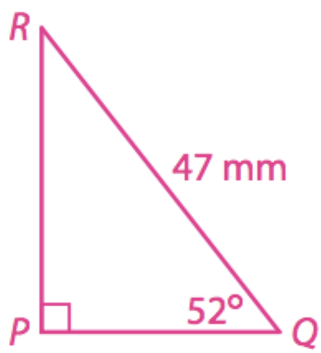
$\sin^{-1} \sin B = \sin^{-1} \frac{8}{12}$

$B = \sin^{-1}\left(\frac{8}{12}\right)$

$180 - 90 - 42 =$

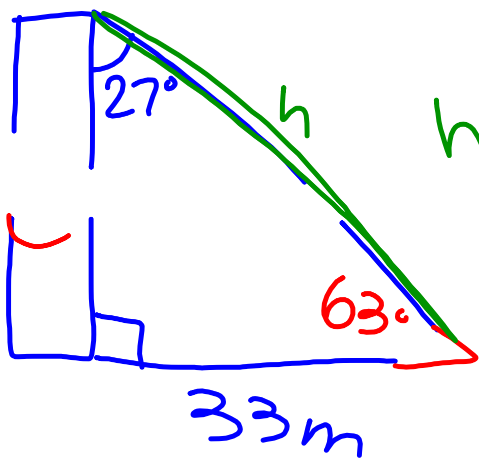
Your Turn!

Solve each right triangle. Round lengths to the nearest tenth and angles to the nearest degree.



A building casts a 33-m shadow when the Sun is at an angle of 27° to the vertical. How tall is the building, to the nearest meter? How far is it from the top of the building to the tip of the shadow? What angle does a ray from the Sun along the edge of the shadow make with the ground?

$180 - 90 - 27 =$



$$\frac{h \cdot \sin 27^\circ}{\sin 27^\circ} = \frac{33 \cancel{h}}{\cancel{h}\,\sin 27^\circ}$$

$$\cos 63^\circ = \frac{33}{h}$$

A shelf extends perpendicularly 7 in. from a wall. You want to place a 9-in. brace under the shelf, as shown. To the nearest tenth of an inch, how far below the shelf will the brace be attached to the wall? To the nearest degree, what angle will the brace make with the shelf and with the wall?