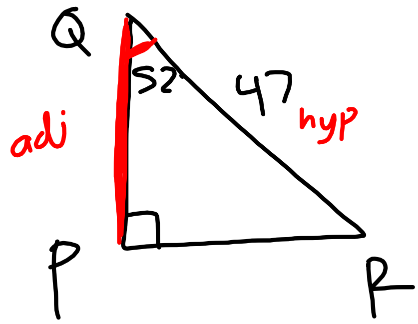
5. back



$P = 90°$

$Q = 52°$

$R = 38°$

$p = 47$

$q =$

$r = 28.9$

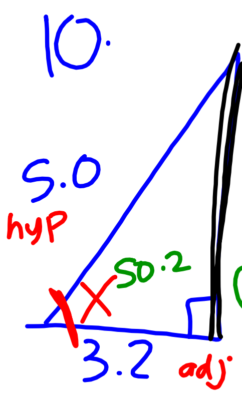
$47 \cdot \cos 52° = \frac{r}{\cancel{47}} \cdot \cancel{47}$

$47 \cdot \cos 52° = r$

$a^2 + b^2 = c^2$

$a^2 + 28.9^2 = 47^2$

10.



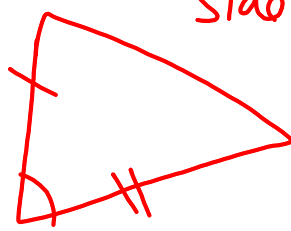
$a^2 + 3.2^2 = 5^2$

$\cos^{-1} \cos X = \cos^{-1} \frac{3.2}{5.0}$

$X = \cos^{-1}\left(\frac{3.2}{5.0}\right)$

Area of non-right Δ:

$$A = \frac{1}{2} \cdot \underbrace{a \cdot b}_{\text{sides}} \cdot \sin \underset{\text{angle}}{C}$$



10-2 Law of Sines

Objectives: Solve non-right triangles

1. I can derive the law of sines using the area of a triangle.

2. I can solve a triangle using the law of sines.

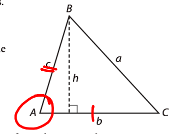
3. I can identify 2 possible triangles and solve.

Explore — Use an Area Formula to Derive the Law of Sines

Recall that the area of a triangle can be found using the sine of one of the angles.

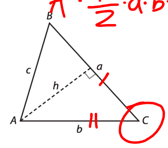
Area $= \frac{1}{2} b \cdot c \cdot \sin(A)$

You can write variations of this formula using different angles and sides from the same triangle.



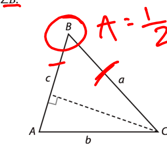
Ⓐ Rewrite the area formula using side length a as the base of the triangle and $\angle C$.

$A = \frac{1}{2} \cdot a \cdot b \cdot \sin C$



Ⓑ Rewrite the area formula using side length c as the base of the triangle and $\angle B$.

$A = \frac{1}{2} a \cdot c \sin B$



Ⓒ What do all three formulas have in common?

find the same area

Ⓓ Why is this statement true?

$\frac{1}{2} b \cdot c \cdot \sin(A) = \frac{1}{2} a \cdot b \cdot \sin(C) = \frac{1}{2} c \cdot a \cdot \sin(B)$

Ⓔ Multiply each area by the expression $\boxed{\frac{2}{abc}}$. Write an equivalent statement.

$\frac{\cancel{2}\cancel{b}\cancel{c}\sin A}{\cancel{2}a\cancel{b}\cancel{c}} = \frac{\cancel{2}\cancel{a}\cancel{b}\sin C}{\cancel{2}\cancel{a}\cancel{b}c}$

$\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$

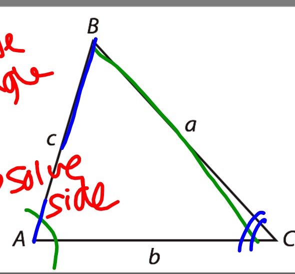
Law of Sines

Law of Sines

Given: $\triangle ABC$

$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ → solve angle

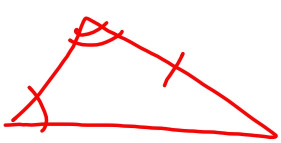
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ → solve side



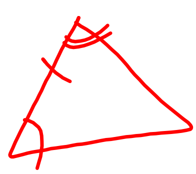
largest side ↔ angle

"butt case"

★ AAS

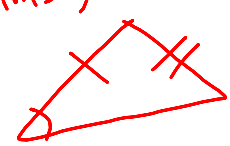


★ ASA



ASS

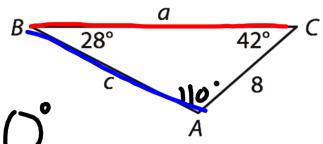
ambiguous case



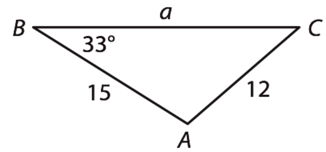
Your Turn

Find all the unknown measures using the given triangle. Round to the nearest tenth.

4.



5.



$A = 110^\circ$

$B = 28^\circ$

$C = 42^\circ$

$a = 16.0$

$b = 8$

$c = 11.4$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\cancel{\sin 110^\circ} \cdot \frac{a}{\cancel{\sin 110^\circ}} = \frac{8}{\sin 28^\circ} \cdot \sin 110^\circ$$

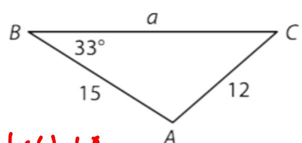
$$a = \frac{8}{\sin(28)} \cdot \sin(110)$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 42^\circ} = \frac{8}{\sin 28^\circ}$$

$c =$

5.



$A = 104.1^\circ$

$B = 33^\circ$

$C = 42.9$

$a = 21.4$

$b = 12$

$c = 15$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\cancel{15} \cdot \frac{\sin C}{\cancel{15}} = \frac{\sin 33^\circ}{12} \cdot 15$$

$$\sin^{-1} \sin C = \sin^{-1}\left(\frac{\sin(33)}{12} \cdot 15\right)$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin 104.1 \cdot \frac{a}{\sin 104.1} = \frac{12}{\sin 33} \cdot \sin 104.1$$

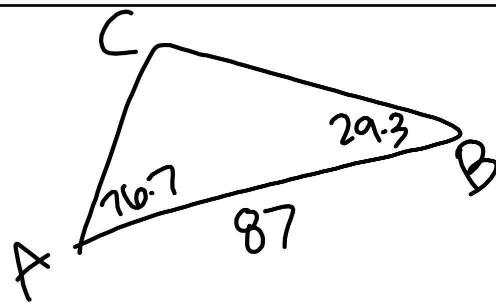
Solve the triangle given:

A = 76.7°

B = 29.3°

~~c=87~~

C = 74°



• a = 88.1

• b = 44.3

c = 87

$$\frac{a}{\sin 76.7} = \frac{87}{\sin 74}$$

$$\frac{b}{\sin 29.3} = \frac{88.1}{\sin 76.7}$$

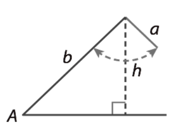
Explain 2 Evaluating Triangles When SSA is Known Information

When you use the Law of Sines to solve a triangle for which you know side-side-angle (SSA) information, zero, one, or two triangles may be possible. For this reason, SSA is called the ambiguous case.

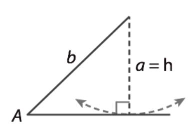
Ambiguous Case

Given a, b, and m$\angle A$.

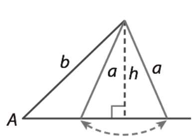
$\angle A$ is acute.



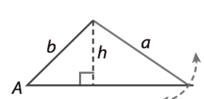
$a < h$
No triangle



$a = h$
One triangle

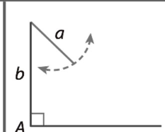


$h < a < b$
Two triangles

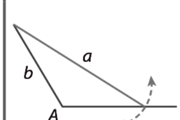


$a \geq b$
One triangle

$\angle A$ is right or obtuse.

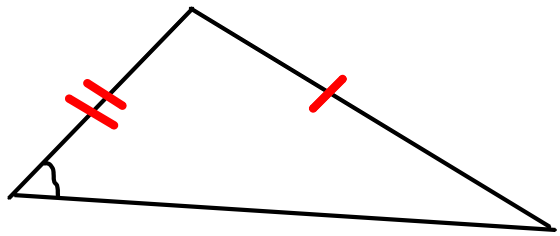


$a \leq b$
No triangle

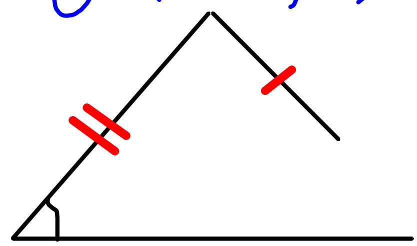


$a > b$
One triangle

1 triangle

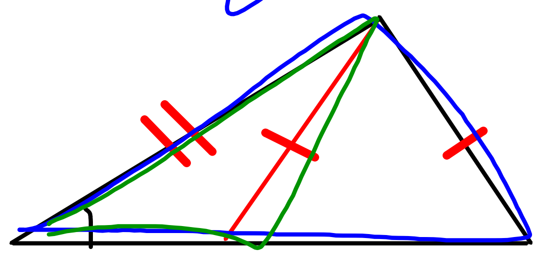


0 triangles

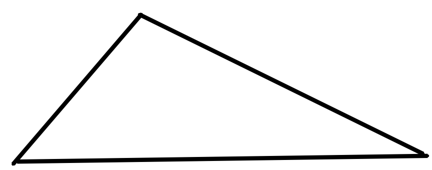


ASS

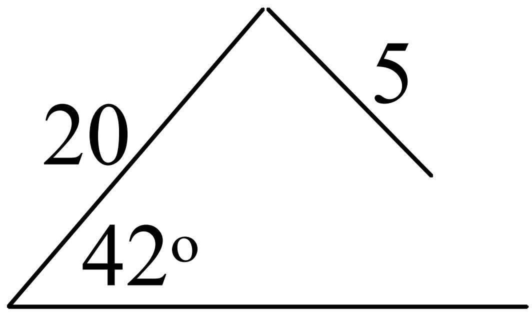
2 triangles



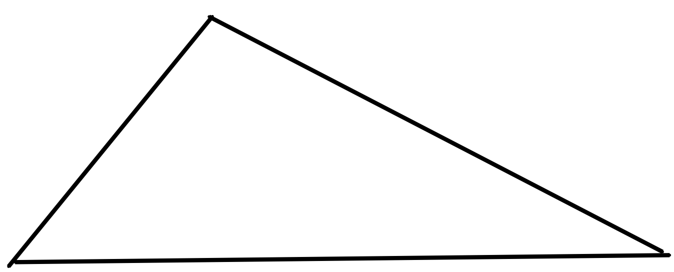
Given a=20, b=5, B=42°



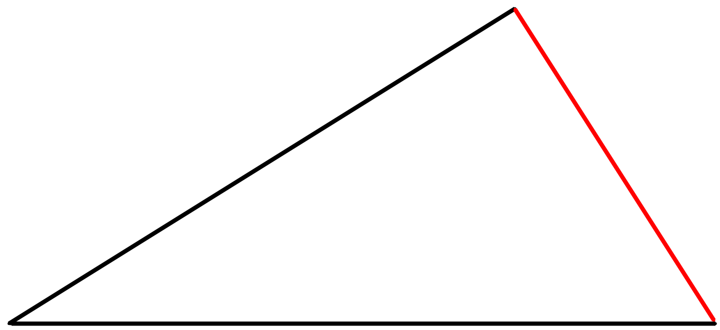
0 Triangles



Given: a=3, b=2, A=40°



Given: a=6, b=8, A=35°



Given: a=37, b=40, A=71°