Announcements

Turn in Golden Tickets (light yellow, "Q2")

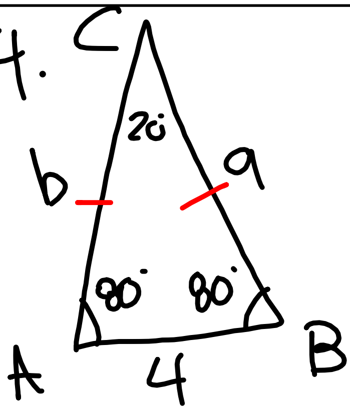
Clean out folders

ALL Q3 work due TODAY

Rev → Fri

Test → Tuesday!

14.



$$C = 20^\circ$$

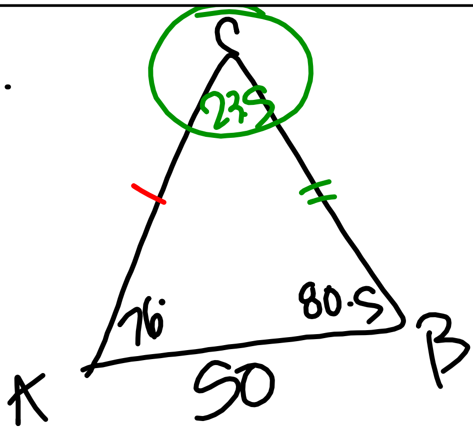
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\cancel{\sin 80^\circ} \cdot \frac{b}{\cancel{\sin 80^\circ}} = \frac{4}{\sin 20^\circ} \cdot \sin 80^\circ$$

$$b = 11.5$$

$$a = 11.5$$

19.



b =

a =

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 80.5} = \frac{50}{\sin 23.5}$$

$$\frac{a}{\sin 76} = \frac{50}{\sin 23.5}$$

Set-up → you solve!

10-3 Law of Cosines

I can solve a triangle using the Law of Cosines.

Area of non-right Δ: $A = \frac{1}{2}\overbrace{ab}^{\text{sides}}\sin \overset{\text{angle}}{C}$

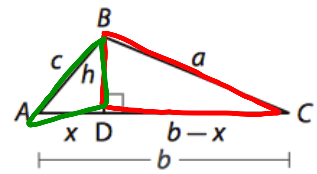
Law of sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ → side

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ → angle

Explore Deriving the Law of Cosines

You learned to solve triangle problems by using the Law of Sines. However, the Law of Sines cannot be used to solve triangles for which side-angle-side (SAS) or side-side-side (SSS) information is given. Instead, you must use the Law of Cosines.

To derive the Law of Cosines, draw $\triangle ABC$ with altitude $\overline{BD}$. If x represents the length of $\overline{AD}$, the length of $\overline{DC}$ is $b - x$.



(A) Use the Pythagorean Theorem to write a relationship for the side lengths of $\triangle BCD$ and for the side lengths of $\triangle ABD$.

$x^2 + h^2 = c^2$

$h^2 + (b-x)^2 = a^2$

$h^2 + (b-x)(b-x) = a^2$

(B) Notice that c^2 is equal to a sum of terms in the equation for a^2. Substitute c^2 for those terms.

$a^2 = h^2 + b^2 - 2bx + x^2$

$a^2 = c^2 + b^2 - 2bx$

(C) In $\triangle ABD$, $\cos A = \frac{x}{c}$. Solve for x. Then substitute into the equation you wrote for a^2.

$c \cdot \cos A = \frac{x}{c} \cdot c$ (adj / hyp), or $x =$ $c \cdot \cos A$.

$a^2 = b^2 -$ ______________ $+ c^2$

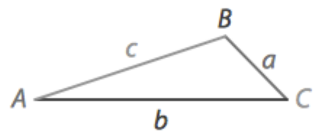
$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

Reflect

1. The equation you wrote in Step D is the Law of Cosines, which is usually written as $a^2 = b^2 + c^2 - 2bc\cos A$. Write formulas using $\cos B$ or $\cos C$ to describe the same relationships in this triangle.

To find the missing side length of a right triangle, you can use the Pythagorean Theorem. To find a missing side length of a general triangle, you can use the Law of Cosines.

Law of Cosines
For $\triangle ABC$, the Law of Cosines states that $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$

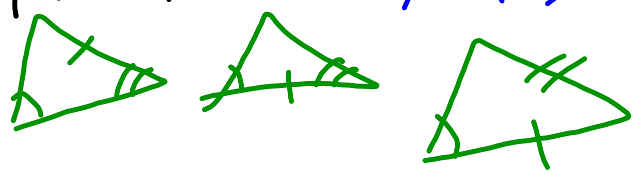


Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

Use: SSS, SAS

Law of Sines → AAS, ASA, ASS

angles



Law of Cosines → SSS, SAS

sides

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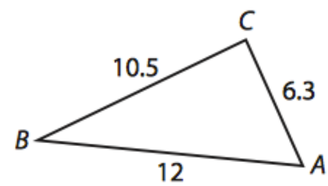
(B) **Step 1** Find the measure of the largest angle, $\angle C$.



$\underline{\qquad\qquad\qquad\qquad\qquad}$	Law of Cosines
$\underline{\qquad}^2 = \underline{\qquad}^2 + \underline{\qquad}^2 - 2\underline{\qquad\qquad\qquad}\cos C$	Substitute.
$\cos C \approx \underline{\qquad\qquad}$	Solve for $\cos C$.
$m\angle C \approx \cos^{-1}(\underline{\qquad\qquad}) \approx \underline{\qquad\quad}$	Solve for $m\angle C$.

Step 2 Find another angle measure.

$b^2 = a^2 + c^2 - 2ac\cos B$	$\underline{\qquad\qquad\qquad\qquad}$
$\underline{\qquad}^2 = \underline{\qquad}^2 + \underline{\qquad}^2 - 2(\underline{\qquad\quad})(\underline{\qquad})\cos B$	Substitute.
$\cos B \approx \underline{\qquad\qquad}$	Solve for $\underline{\qquad\qquad}$.
$m\angle B \approx \cos^{-1}(\underline{\qquad\qquad\qquad}) \approx \underline{\qquad\quad}$	Solve for $\underline{\qquad\qquad}$.

Step 3 Find the third angle measure.

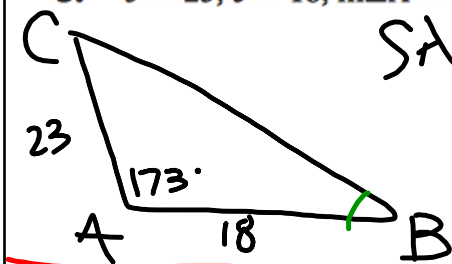
$m\angle A + \underline{\qquad\quad} + \underline{\qquad\quad} = 180°$	$\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$
$m\angle A = \underline{\qquad\quad}$	Solve for $m\angle A$.

Your Turn

Solve △*ABC*. Round to the nearest tenth.

3. $b = 23$, $c = 18$, $m\angle A = 173°$

4. $a = 35$, $b = 42$, $c = 50.3$



SAS → L of C

$A = 173°$
$B = 4.4°$
$C = 2.6°$
$a = 40.9$
$b = 23$
$c = 18$

$$a^2 = b^2 + c^2 - \overbrace{2bc\cos A}^{\text{mult.}}$$

$$a^2 = 23^2 + 18^2 - 2(23)(18)\cdot\cos 173$$

$$\sqrt{a^2} = \sqrt{1674.8}$$

$$\boxed{a = 40.9}$$

$$b^2 = a^2 + c^2 - 2\cdot a\cdot c\cdot\cos B$$

$$23^2 = 40.9^2 + 18^2 - 2(40.9)(18)\cdot\cos B$$

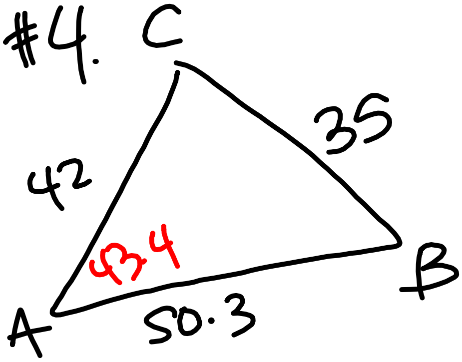
calc:

$$(23^2 - 40.9^2 - 18^2) \div (-2\cdot 40.9\cdot 18)$$

$$\cos^{-1} 0.997 = \cancel{\cos^{-1}}\cancel{\cos} B$$

$$4.4° = B$$

#4.



$a = 35$
$b = 42$
$c = 50.3$

$A = 43.4°$
$B = 55.6°$
$C = 81°$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$35^2 = 42^2 + 50.3^2 - 2(42)(50.3)\cos A$$

calc → $(35^2 - 42^2 - 50.3^2) \div (-2 \cdot 42 \cdot 50.3) =$

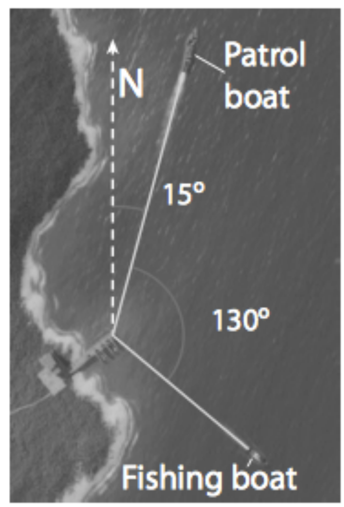
$$\cos^{-1} 0.726 = \cos^{-1} \cos A$$

$$A = 43.4°$$

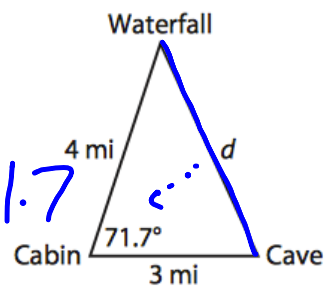
$$(42^2 - 35^2 - 50.3^2) \div (-2 \cdot 35 \cdot 50.3)$$

$$\cos^{-1} =$$

A coast guard patrol boat and a fishing boat leave a dock at the same time at the courses shown. The patrol boat travels at a speed of 12 nautical miles per hour (12 knots), and the fishing boat travels at a speed of 5 knots. After 3 hours, the fishing boat sends a distress signal picked up by the patrol boat. If the fishing boat does not drift, how long will it take the patrol boat to reach it at a speed of 12 knots?



If Lucas hikes at an average of 2.5 miles per hour, how long will it take him to travel from the cave to the waterfall? Round to the nearest tenth of an hour.



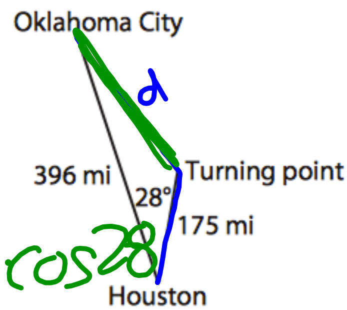
$d^2 = 4^2 + 3^2 - 2(4)(3)\cdot\cos 71.7$

$\sqrt{d^2} = \sqrt{17.5}$

$d = 4.2$

$\frac{4.2}{2.5} = \boxed{1.7 \text{ hrs}}$

A pilot is flying from Houston to Oklahoma City. To avoid a thunderstorm, the pilot flies 28° off of the direct route for a distance of 175 miles. He then makes a turn and flies straight on to Oklahoma City. To the nearest mile, $\boxed{\text{how much farther than}}$ the direct route was the route taken by the pilot?



$$d^2 = 396^2 + 175^2 - 2(396)(175)\cos 28$$

$$\sqrt{d^2} = \sqrt{65064.5}$$

$$d = 255$$

Traveled → $175 + 255 = 430$

Direct → 396

$430 - 396 = \boxed{34 \text{ extra miles}}$

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