

Radical Review

Definition
nth root

$$\sqrt[n]{b} = a \text{ means } b = a^n$$

- if $n \geq 2$ and even then a and b must be greater than or equal to 0.
- if $n \geq 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:

The symbol $\sqrt{\quad}$ is called the radical

n is called the index

b is called the radicand

if there is no index, it is 2

$$\sqrt[2]{4}$$

Know your powers and roots

$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$

Evaluate

$$\sqrt{9}$$

3

$$\sqrt{49}$$

7

$$\sqrt[4]{16}$$

- 2

$$\sqrt[3]{64}$$

4

$$\sqrt[3]{-8}$$

-2 · -2 · -2

-2

$$\sqrt[4]{81}$$

3

Simplifying

If $n \geq 2$ is a positive integer and a is a real number, then

~~$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$~~

~~$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$~~

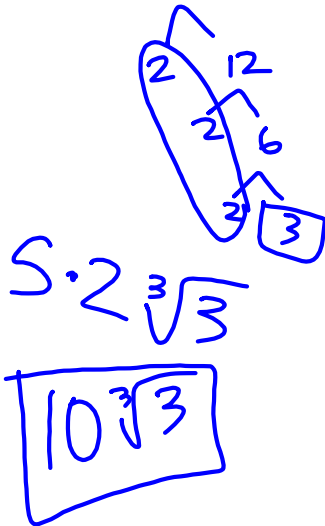
- ① pulling out a variable
- ② index even $\sqrt{\#}$
- ③ odd exponent \rightarrow $| \text{abs val.} |$

$$(-3)(-3) = 9 \quad (3)(3) = 9$$

Simplify

(remember $\sqrt{x^2} = |x|$)

$$5 \cdot \sqrt[3]{24}$$



$$\sqrt{128x^2}$$

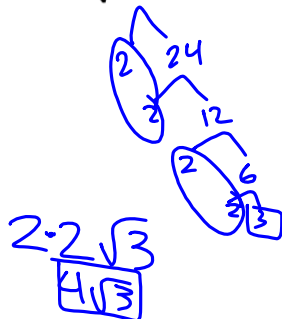


$$\sqrt[4]{20}$$

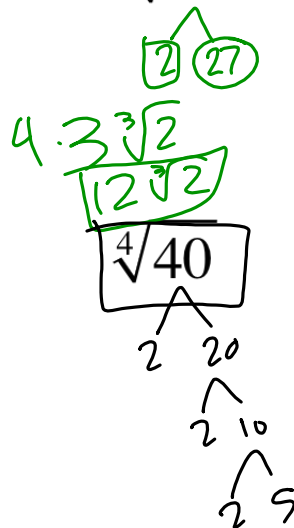


You try

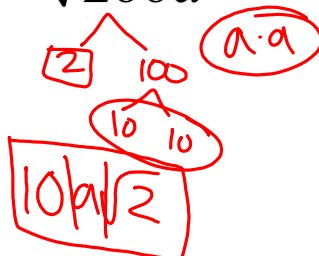
$$\sqrt{48}$$



$$4 \sqrt[3]{54}$$

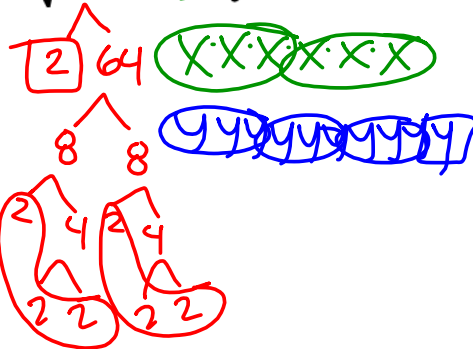


$$\sqrt{200a^2}$$



You Try

$$\sqrt[3]{128x^6y^{10}}$$



$$2 \cdot 2 \cdot 2 \cdot 4 \cdot 4 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$4x^2y^3\sqrt[3]{2y}$$

$$\sqrt[4]{16a^5b^{11}}$$

$$2ab^2\sqrt[4]{ab^3}$$

Raise each of the following to the $\frac{1}{2}$ power.

1, 4, 9, 16, 25, 36

↓ ↓ ↓
1, 2, 3, 4, 5, 6

$$a^{\left(\frac{1}{2}\right)} = \sqrt{a}$$

Raise each of the following to the $\frac{1}{3}$ power.

1, 8, 27, 64, 125, 216
1 2 3 4 5 6

$$a^{\left(\frac{1}{3}\right)} = \underline{\sqrt[3]{a}}$$

Fractional exponent

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

n is an integer bigger than or equal to 2

1, 8, 27, 64, 125, 216

$$a^{\left(\frac{2}{3}\right)} = \underline{\sqrt[3]{a^2}} = (\sqrt[3]{a})^2 \quad a^{2 \cdot \frac{1}{3}}$$

$$a^{\left(\frac{m}{n}\right)} = \underline{\sqrt[n]{a^m}} = (\sqrt[n]{a})^m$$

$$* a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

a is real, m/n is a rational number in lowest terms with n bigger or equal to 2

roots in the ground
powers in the sky

Write each of the following as a radical and (simplify, if possible).

$$9^{\frac{1}{2}} \sqrt[2]{9} = \boxed{3}$$

$$(-64)^{\frac{1}{3}} = \sqrt[3]{-64} = \boxed{-4}$$

$$100^{\frac{1}{2}} = \sqrt{100} = \boxed{10}$$

$$-100^{\frac{1}{2}} = -\sqrt{100} = \boxed{-10}$$

$$z^{\frac{1}{2}} = \sqrt{z}$$

$$(-100)^{\frac{1}{2}} = \sqrt{-100}$$

$10 \cdot 10 = 100$
 $-10 \cdot -10 = 100$

$$\boxed{10i}$$

Rewrite in exponent form

$$\sqrt[7]{x} = x^{\frac{1}{7}} \quad \sqrt[4]{b} = b^{\frac{1}{4}}$$

You try

$$\sqrt[12]{r} = r^{\frac{1}{12}} \quad \sqrt[5]{d} = d^{\frac{1}{5}}$$

Adding, Subtracting, and Multiplying Radical expressions

Product Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $n \geq 2$ is an integer, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$\sqrt[3]{x} \cdot \sqrt[3]{x}$

 $\sqrt[3]{x \cdot y}$
 $\sqrt[3]{x} \cdot \sqrt[3]{y}$

Multiply and Simplify Assuming all variables are greater than or equal to zero. *no abs value.*

$$\sqrt{3} \cdot \sqrt{15}$$

$$\sqrt{3 \cdot 15}$$

$$\sqrt{45}$$

$$3\sqrt{5}$$

$$\sqrt[3]{4x} \cdot \sqrt[3]{2x^4}$$

$$\sqrt[3]{8x^5}$$

$$2x \sqrt[3]{x}$$

$$6x \sqrt[3]{x}$$

$$\sqrt[4]{27a^2b^5} \cdot \sqrt[4]{6a^3b^6}$$

$$\sqrt[4]{27a^2b^5 \cdot 6a^3b^6} \quad a \cdot a \cdot a \cdot a$$

$$\sqrt[4]{162a^5b^{11}}$$

$$\sqrt[4]{81} = 3$$

$$\sqrt[4]{b^4} = b$$

$$\sqrt[4]{2ab^3}$$

$$\frac{11}{7} - 4 = \frac{11 - 28}{7} = -\frac{17}{7}$$

Quotient Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, $b \neq 0$, $n \geq 2$ is an integer, then

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Simplify Assuming all variables are greater than or equal to zero.

$$\frac{\sqrt{24a^3}}{\sqrt{6a}} = \sqrt{\frac{24a^3}{6a}} = \sqrt{4a^2} = 2a$$

$$\frac{-2\sqrt[3]{54a}}{\sqrt[3]{2a^4}} = -2 \frac{\sqrt[3]{34a}}{\sqrt[3]{2a^4}} = -2 \sqrt[3]{\frac{27}{a^3}}$$

$$-2 \cdot \frac{\sqrt[3]{27}}{\sqrt[3]{a^3}} = \frac{-2 \cdot 3}{a} = \boxed{\frac{-6}{a}}$$

Add or subtract as indicated. Assume all variables are real numbers greater than or equal to zero *-no abs value*

$$3x\sqrt{20x} - 7\sqrt{5x^3}$$

$6x\sqrt{5x} - 7x\sqrt{5x}$ $-x\sqrt{5x}$

(Handwritten notes: 2, 10, 4, 5, $x \cdot x \cdot x$)

$$3\sqrt{5} + 7\sqrt{13}$$

$3x + 7y$

$$4\sqrt{14} - 3\sqrt{8}$$

$$-5x\sqrt[3]{54x} + 7\sqrt[3]{2x^4}$$

like terms

$$5\sqrt{x} - 3\sqrt{x}$$

$$5x - 3x$$

$$2x$$

$$2\sqrt{x}$$

$$11\sqrt{3xy} + 12\sqrt{3xy}$$

$$= 23\sqrt{3xy}$$

$$5x\sqrt{2} - 3x\sqrt{2}$$

$$2x\sqrt{2}$$

March 26, 2015

