Radical Review

Definition
nth root

$$\sqrt[n]{b} = a \quad \text{means} \quad b = a^n$$

- if $n \geq 2$ and even then a and b must be greater than or equal to 0.
- if $n \geq 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:

The symbol $\sqrt{\ }$ is called the radical

n is called the index

b is called the radicand

if there is no index, it is 2

$\sqrt[2]{4}$

Know your powers and roots

$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$

Evaluate

$\sqrt{9}$ — 3

$\sqrt{49}$ — 7

$\sqrt[4]{16}$ — 4 4 — 2 2 2 2 — $= \boxed{2}$

$\sqrt[3]{64}$ — 4

$\sqrt[3]{-8}$ — $-2 \cdot -2 \cdot -2$ — $\boxed{-2}$

$\sqrt[4]{81}$ — 9 9 — 3 3 3 3 — $\boxed{3}$

Simplifying

If n≥2 is a positive integer and a is a real number, then

$\sqrt[n]{a^n} = a \quad$ if $n \ge 3$ is odd

$\sqrt[n]{a^n} = |a| \quad$ if $n \ge 2$ is even

① pulling out a variable

② index even $\sqrt[\#]{\ }$

③ odd exponent → | | abs val.

$(-3)(-3) = 9 \qquad (3)(3) = 9$

Simplify

(remember $\sqrt{x^2} = |x|$)

$5\cdot\sqrt[3]{24}$

2 12

2 6

2 $\boxed{3}$

$5\cdot 2\sqrt[3]{3}$

$\boxed{10\sqrt[3]{3}}$

$\boxed{\sqrt[4]{20}}$

5 4

2 2

$\sqrt[2]{128x^2}$

$\boxed{2}$ 64 $x\cdot x$

8 8

$\boxed{8|x|\sqrt{2}}$

You try

$\sqrt{48}$

2 24

2 12

2 6

3 $\boxed{3}$

$2\cdot 2\sqrt{3}$

$\boxed{4\sqrt{3}}$

$4\cdot\sqrt[3]{54}$

$\boxed{2}$ 27

$4\cdot 3\sqrt[3]{2}$

$\boxed{12\sqrt[3]{2}}$

$\sqrt[2]{200a^2}$

$\boxed{2}$ 100 $a\cdot a$

10 10

$\boxed{10|a|\sqrt{2}}$

$\boxed{\sqrt[4]{40}}$

2 20

2 10

2 5

You Try

$$\sqrt[3]{128x^6y^{10}}$$

$2 \quad 64$

$8 \quad 8$

$2 \quad 4 \quad 2 \quad 4$

$2 \quad 2 \quad 2 \quad 2$

$x \cdot x \cdot x \cdot x \cdot x \cdot x$

$yyyyyyyyyy$

$2 \cdot 2\, x\, x\, y\, y\, y \sqrt[3]{2y}$

$$\boxed{4x^2y^3\sqrt[3]{2y}}$$

$$\sqrt[4]{16a^5b^{11}}$$

$$\boxed{2ab^2\sqrt[4]{ab^3}}$$

Raise each of the following to the $\frac{1}{2}$ power.

$$1,\ 4,\ 9,\ 16,\ 25,\ 36$$

$$1, 2, 3, 4, 5, 6$$

$$a^{\left(\frac{1}{2}\right)} = \sqrt[2]{a}$$

Raise each of the following to the $\frac{1}{3}$ power.

$$1, 8, 27, 64, 125, 216$$

1 2 3 4 5 6

$$a^{\left(\frac{1}{3}\right)} = \underline{\sqrt[3]{a}}$$

Fractional exponent

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

n is an integer bigger then or equal to 2

1, 8, 27, 64, 125, 216

$$a^{\left(\frac{2}{3}\right)} = \underline{\sqrt[3]{a^2}} = \left(\sqrt[3]{a}\right)^2 \quad a^{\frac{1}{2}} \ldots \sqrt[2]{\ }$$

$$a^{\left(\frac{m}{n}\right)} = \underline{\sqrt[n]{a^m}} = \left(\sqrt[n]{a}\right)^m$$

$$* \quad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

a is real, m/n is a rational number in lowest terms with n bigger or equal to 2

roots in the ground
powers in the sky

Write each of the following as a radical and simplify, if possible.

$9^{\frac{1}{2}}$ $\sqrt[2]{9^1} = \boxed{3}$

$(-64)^{\frac{1}{3}}$ $\sqrt[3]{-64^1} = \boxed{-4}$

$100^{\frac{1}{2}}$

$\sqrt{100}$

$\boxed{10}$

$-100^{\frac{1}{2}}$

$-\sqrt{100}$

$\boxed{-10}$

$(-100)^{\frac{1}{2}}$

$\sqrt{-100}$

$10 \cdot 10 = 100$

$-10 \cdot -10 = 100$

$\boxed{10i}$

$z^{\frac{1}{2}}$

$\sqrt[2]{z^1}$

Rewrite in exponent form

$\sqrt[7]{x^1} = x^{\frac{1}{7}}$

$\sqrt[4]{b}$ $b^{\frac{1}{4}}$

You try

$\sqrt[12]{r}$ $r^{\frac{1}{12}}$

$\sqrt[5]{d}$ $d^{\frac{1}{5}}$

Adding, Subtracting, and Multiplying Radical expressions

Product Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $n \geq 2$ is an integer, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$\sqrt[2]{x} \cdot \sqrt[3]{x}$

$\sqrt[3]{x \cdot y}$

$\sqrt[3]{x} \cdot \sqrt[3]{y}$

Multiply and Simplify Assuming all variables are greater then or equal to zero. → no abs value.

$\sqrt[2]{3} \cdot \sqrt[2]{15}$

$\sqrt[2]{3 \cdot 15}$

$\sqrt[2]{45}$

3 15

3 5

$\boxed{3\sqrt{5}}$

$3\sqrt[3]{4x} \cdot \sqrt[3]{2x^4}$

$3\sqrt[3]{8x^5}$

$3 \cdot 2x\sqrt[3]{x}$

$\boxed{6x\sqrt[3]{x}}$

$\sqrt[4]{27a^2b^5} \cdot \sqrt[4]{6a^3b^6}$

$\sqrt[4]{27a^2b^5 \cdot 6a^3b^6}$ a·a·a·a·a

$\sqrt[4]{162a^5b^{11}}$

2 81

9 9

3 3 3 3

$\boxed{3ab^2\sqrt[4]{2ab^3}}$

11
-4
7
-4
3

Quotient Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, $b \neq 0$, $n \geq 2$ is an integer, then

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Simplify Assuming all variables are greater then or equal to zero.

$$\frac{\sqrt[2]{24a^3}}{\sqrt[2]{6a}} = \sqrt{\frac{24a^3}{6a}} = \sqrt{4a^2}$$

$$\boxed{2|a|}$$

$$\frac{\cancel{a} \cdot a \cdot a}{\cancel{a}}$$

$$\frac{-2\sqrt[3]{54a}}{\sqrt[3]{2a^4}}$$

$$-2\sqrt[3]{\frac{54a}{2a^4}} = -2\sqrt[3]{\frac{27}{a^3}}$$

$$-2 \cdot \frac{\sqrt[3]{27}}{\sqrt[3]{a^3}}$$

$$\frac{-2 \cdot 3}{a} = \boxed{\frac{-6}{a}}$$

Add or subtract as indicated. Assume all variables are real numbers greater then or equal to zero → no abs value

$3x\sqrt{20x} - 7\sqrt{5x^3}$

$x \cdot x \cdot x$

$6x\sqrt{5x} - 7x\sqrt{5x}$

$\boxed{-x\sqrt{5x}}$

$\boxed{3\sqrt{5} + 7\sqrt{13}}$

1 5 1 13

$3x + 7y$

$4\sqrt{14} - 3\sqrt{8}$

$-5x\sqrt[3]{54x} + 7\sqrt[3]{2x^4}$

like terms

$5\sqrt{x} - 3\sqrt{x}$

$5x - 3x$

$2x$

$2\sqrt{x}$

$11\sqrt{3xy} + 12\sqrt{3xy} = 23\sqrt{3xy}$

$5x\sqrt{2} - 3x\sqrt{2}$

$\boxed{2x\sqrt{2}}$