11-3 Solving Radical Equations

$\sqrt[3]{x}^3 = 2^3$ $\quad$ $\sqrt{x}^2 = 4^2$

$x = 8$ $\quad$ $x = 16$

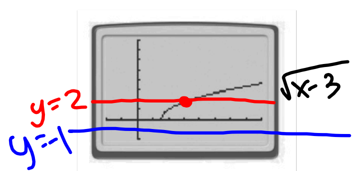
Objectives:

1. I can solve radical equations and check for extraneous solutions.

2. I can manipulate literal equations.

Remember that you can graph the two sides of an equation as separate functions to find solutions of the equation: a solution is any x-value where the two graphs intersect.

The graph of $y = \sqrt{x-3}$ is shown on a calculator window of $-4 \leq x \leq 16$ and $-2 \leq y \leq 8$. Reproduce the graph on your calculator. Then add the graph of $y = 2$.



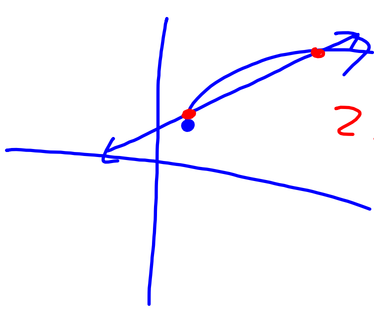
y_1 $\quad$ y_2

How many solutions does the equation $\sqrt{x-3} = 2$ have? 1 soln How do you know?

On your calculator, replace the graph of $y = 2$ with the graph of $y = -1$.

How many solutions does the equation $\sqrt{x-3} = -1$ have? ______ How do you know?

no soln



2 solns

Graph both sides of $\sqrt{4x-4} = x + 1$ as separate functions on your calculator.

How many solutions does $\sqrt{4x-4} = x + 1$ have? ______

Replace the graph of $y = x + 1$ with the graph of $y = \frac{1}{2}x$.

How many solutions does $\sqrt{4x-4} = \frac{1}{2}x$ have? ______

Replace the graph of $y = \frac{1}{2}x$ with the graph of $y = 2x - 5$.

How many solutions does $\sqrt{4x-4} = 2x - 5$ have? ______

Example 1 **Solve the equation. Check for extraneous solutions.**

(A) $2 + \sqrt{x+10} = x$

$-2 \qquad -2$

$(\sqrt{x+10})^2 = (x-2)^2$

$* \; x+10 = (x-2)(x-2)$

$x + 10 = x^2 - 2x - 2x + 4$

$-x - 10 \qquad -x \qquad -10$

$0 = 1x^2 \boxed{-5}x - 6$

$(x+1)(x-6)$

$x+1=0 \qquad x-6=0$

$\cancel{x=-1} \qquad \boxed{x=6}$

$x^2 \to$ ① quad. formula
② factor

$1 \cdot -6 = -6 \qquad -5$

$+1 - 6$

$2 \quad 3$

$2 + \sqrt{x+10} = x$

$2 + \sqrt{-1+10} = -1$

$2 + \sqrt{9} = -1$

$2 + 3 = -1$

$5 = -1$

$2 + \sqrt{6+10} = 6$

$2 + 4 = 6$ ✓

(B) $(x+6)^{\frac{1}{2}} - (2x-4)^{\frac{1}{2}} = 0$ $\rightarrow (\sqrt{x+6}-\sqrt{2x-4})(\sqrt{x+6}-\sqrt{2x-4})$

$(\sqrt{x+6} - \sqrt{2x-4})^2 = (0)^2$

$+\sqrt{2x-4} \quad +\sqrt{2x-4}$

$(\sqrt{x+6})^2 = (\sqrt{2x-4})^2$

$x+6 = 2x-4$

$-6 \qquad -6$

$x = 2x - 10$

$-2x \quad -2x$

$\frac{-x}{-1} = \frac{-10}{-1}$

$\boxed{x = 10}$

$\sqrt{10+6} - \sqrt{20-4} = 0$

$4 - 4 = 0$ ✓

6. Solve $(x+5)^{\frac{1}{2}} - 2 = 1.$ check for extraneous soln.

PLUG in answers!

$\sqrt{x+5} - 2 = 1$

$+2 \quad +2$

$(\sqrt{x+5})^2 = (3)^2$

$x+5 = 9$

$-5 \quad -5$

$\boxed{x = 4}$

$\sqrt{4+5} - 2 = 1$

$\sqrt{9} - 2 = 1$

$3 - 2 = 1$

$1 = 1$ ✓

Solve the following, check for extraneous solutions

$\frac{\cancel{2}\sqrt{x}}{2} = \frac{3\sqrt{x-2}}{2}$

$(\sqrt{x})^2 = \left(\frac{3}{2}\sqrt{x-2}\right)^2 \rightarrow \left(\frac{3}{2}\cdot\sqrt{x-2}\right)\left(\frac{3}{2}\cdot\sqrt{x-2}\right)$

$\frac{3}{2}\cdot\frac{3}{2}\cdot\sqrt{x-2}\cdot\sqrt{x-2}$

$(\sqrt{x-2})^2$

$x = \frac{9}{4}\left(\frac{x-2}{1}\right)$

$x = \frac{9}{4}x - \frac{18}{4}$

$-\frac{9}{4}x \quad -\frac{9}{4}x$

$\frac{4}{4}x - \frac{9}{4}x = -\frac{18}{4}$

$\frac{4}{-5}\cdot\frac{-5x}{4} = \frac{+18}{\cancel{4}}\cdot\frac{\cancel{4}}{+5}$

$\boxed{x = \frac{18}{5}}$

$\sqrt{2x+5} + 4 = 3$

$\sqrt{5x-11} = x - 1$

$(2\sqrt{x})^2 = (3\sqrt{x-2})^2$

$(xy)^2 = x^2 \cdot y^2$

$2^2 \cdot \sqrt{x}^2 = 3^2 \cdot \sqrt{x-2}^2$

$4x = 9(x-2)$

$4x = 9x - 18$

$-9x \quad -9x$

$\frac{-5x}{-5} = \frac{-18}{-5}$

$\boxed{x = 18/5}$

$\sqrt{2x+5} + 4 = 3$

$-4 \quad -4$

$*(\sqrt{2x+5})^2 = (-1)^2 \rightarrow \sqrt{\#} = -1$

$2x + 5 = 1$

$-5 \quad -5$

$\frac{2x}{2} = \frac{-4}{2}$

$\cancel{x = -2}$

$\sqrt{-4+5} + 4 = 3$

$1 + 4 = 3 \quad X$

no soln

H

Solve the following, check for extraneous solutions

$$\sqrt{2x+5} = \sqrt{x}+2$$

Example 2 Solve the equation.

(A) $\sqrt[3]{x+2}+7=5$

$-7 \quad -7$

$\left(\sqrt[3]{x+2}\right)^3 = (-2)^3$

$x+2=-8$

$-2 \quad -2$

$\boxed{x=-10}$

$\sqrt[3]{-10+2}+7=5$

$\sqrt[3]{-8}+7=5$

$-2 \quad 4$

$-2 \quad -2$

$-2+7=5$

$5=5$ ✓

(B) $\sqrt[3]{x-5} = x+1$

Your Turn

8. Solve $2(x-50)^{\frac{1}{3}} = -10$.

$$\frac{2\sqrt[3]{x-50}}{2} = \frac{-10}{2}$$

$$\left(\sqrt[3]{x-50}\right)^3 = (-5)^3$$

$$x - 50 = -125$$
$$+50 \qquad +50$$

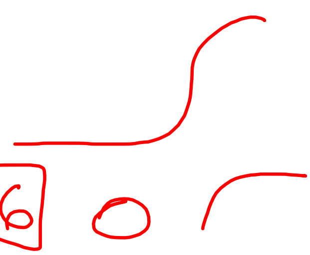
$$x = -75$$

Solve the following:

$$\left(\sqrt[3]{x-5}\right)^3 = \left(\sqrt[3]{7-x}\right)^3$$

$$x - 5 = 7 - x$$

$$2x = 12 \qquad \boxed{x = 6}$$



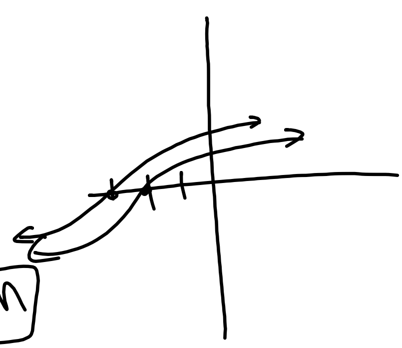
$$\left(\sqrt[3]{x+2}\right)^3 = \left(\sqrt[3]{x+3}\right)^3$$

$$x + 2 = x + 3$$
$$-2 \qquad -2$$

$$x = x + 1$$
$$-x \quad -x$$

$$0 = 1$$

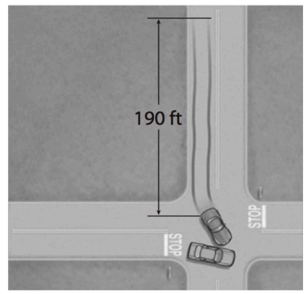
$\boxed{\text{no soln}}$



(A) **Driving** The speed s in miles per hour that a car is traveling when it goes into a skid can be estimated by using the formula $s = \sqrt{30fd}$, where f is the coefficient of friction and d is the length of the skid marks in feet.

After an accident, a driver claims to have been traveling the speed limit of 55 mi/h. The coefficient of friction under the conditions at the time of the accident was 0.6, and the length of the skid marks is 190 feet. Is the driver telling the truth about the car's speed? Explain.

Use the formula to find the length of a skid at a speed of 55 mi/h. Compare this distance to the actual skid length of 190 feet.



Your Turn

length age

9. **Biology** The trunk length (in inches) of a male elephant can be modeled by $l = 23\sqrt[3]{t} + 17$, where t is the age of the elephant in years. If a male elephant has a trunk length of 100 inches, about what is his age?

$$100 = 23\sqrt[3]{t} + 17$$
$$-17 \qquad\qquad -17$$
$$\frac{83}{23} = \frac{23\sqrt[3]{t}}{23}$$
$$3.6^3 = \left(\sqrt[3]{t}\right)^3$$
$$\boxed{47 = t}$$

$$a\sqrt[n]{\boxed{b}(x-h)}+k$$

V.S. neg → x-axis

H.S. neg → y-axis

R or L x's lie

U or D

$\sqrt{x}$

$\sqrt[3]{x}$