12-3 Part C. ✓ part A.

Radian Measure, Arc Length, and Sectors

I can convert between degrees and radians

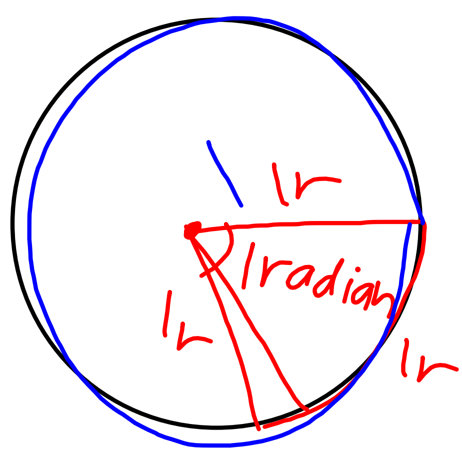
I can find exact trig values

I can find arc lengths

I can find the area of sectors

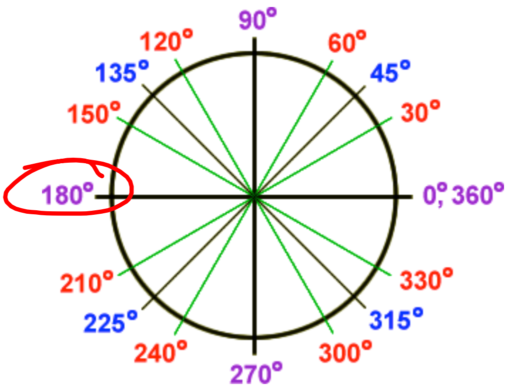
What are radians?

$C = 2\pi r$
$= 2 \cdot \pi \cdot 1$
$= 2\pi$

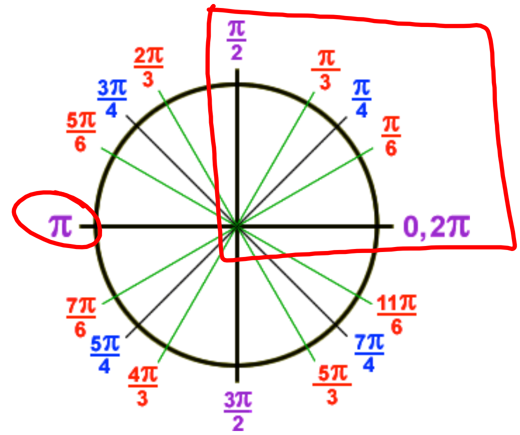


$360°$
2π

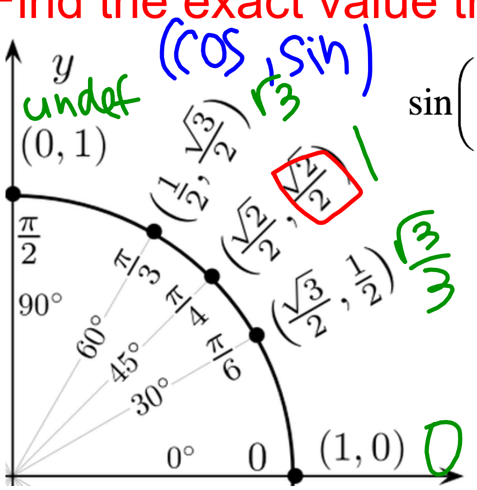
Degrees



Radians



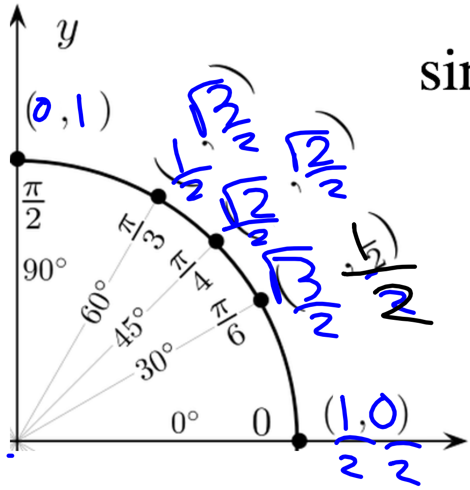
Find the exact value trig values:



tan $\frac{y}{x}$

$\sin\left(\frac{\pi}{4}\right)$ $\boxed{\frac{\sqrt{2}}{2}}$ $\cos\left(\frac{\pi}{6}\right)$ $\boxed{\frac{\sqrt{3}}{2}}$ $\tan\left(\frac{\pi}{2}\right)$ $\boxed{\text{undef}}$

You try: Find the exact value trig values



$\sin(0)$ $\cos\left(\frac{\pi}{3}\right)$ $\tan\left(\frac{\pi}{4}\right)$

$\sin\left(\frac{\pi}{6}\right)$

$1/2$

Converting between degrees and radians:

$$\boxed{x^\circ \bullet \frac{\pi}{180^\circ} = radians \qquad xradians \bullet \frac{180^\circ}{\pi} = \deg rees}$$

OR

$$\frac{rad}{\pi} = \frac{\deg}{180^\circ}$$

Example:

Convert degrees to radians:

a) $\frac{90^\circ}{1} \cdot \frac{\pi}{180^\circ} = \boxed{\frac{90\pi}{180}} = \boxed{\frac{\pi}{2}}$

b) $\frac{135^\circ}{1} \cdot \frac{\pi}{180^\circ} = \boxed{\frac{135\pi}{180}} = \boxed{\frac{3\pi}{4}}$

Convert radians to degrees:

c) $-\frac{3\cancel{\pi}}{4} \cdot \frac{180^\circ}{\cancel{\pi}} = -\frac{540^\circ}{4}$

$\boxed{-135^\circ}$

d) $\frac{16\cancel{\pi}}{9} \cdot \frac{180^\circ}{\cancel{\pi}} = \frac{2880^\circ}{9}$

$= \boxed{320^\circ}$

1. $\frac{60^\circ}{1} \cdot \frac{\pi}{180} = \boxed{\frac{60\pi}{180}} = \frac{\pi}{3}$

6. $\frac{\cancel{\pi}}{2} \cdot \frac{180}{\cancel{\pi}} = \frac{180}{2} = 90^\circ$

11. $\sin\left(\frac{\pi}{2}\right)$

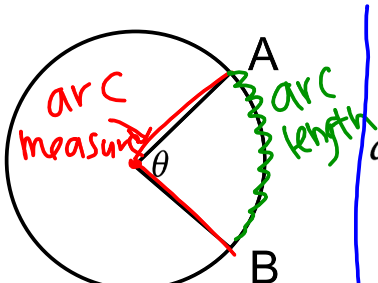
Recall:

Circumference of a circle

$$C = 2\pi r$$

Arc Length:

A proportion of the circumference of the circle. You can use the measure of the arc (in degrees) to find its length (in linear units.)



Degrees

$$arclength = \left(\frac{\overset{\frown}{AB}}{360^o}\right)\underbrace{\boxed{2\pi r}}_{C}$$

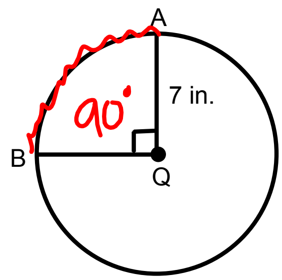
Radians

$$arclength = r\theta$$

in, cm

Example:

Find the length of $\overset{\frown}{AB}$

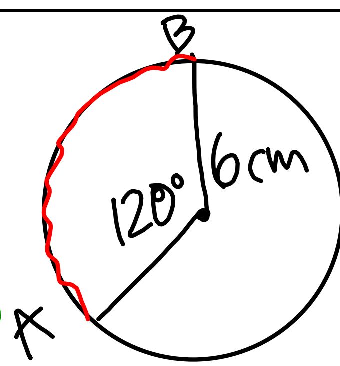


$\frac{\overset{\frown}{AB}}{360°} \cdot 2\pi r$

$\frac{90°}{360°} \cdot \frac{2\pi \cdot 7}{1} = \frac{1260\pi}{360} = \boxed{\frac{7\pi}{2} \text{ in}}$

10.99 in

17.

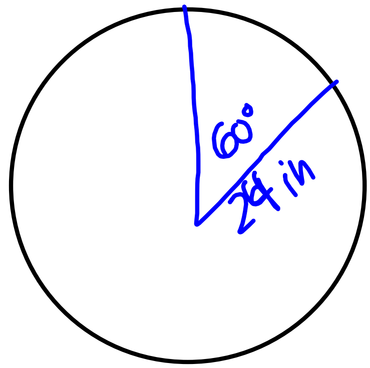


$\frac{120}{360} \cdot 2 \cdot \pi \cdot 6\text{cm}$

4π cm

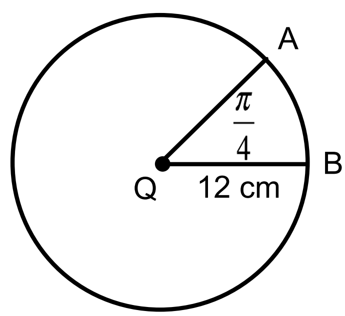
20.

θ angle



Example:

Find the length of $\overset{\frown}{AB}$

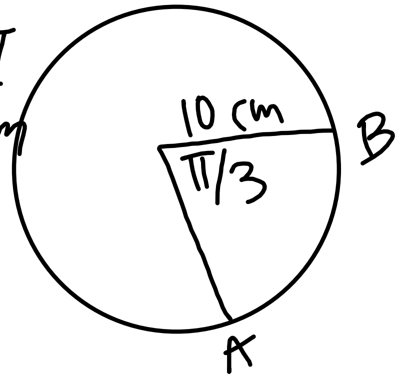


arclength = $r\theta$

radius angle

$$\frac{12}{1}\cdot\frac{\pi}{4} = \frac{12\pi}{4} = \boxed{3\pi \text{ cm}}$$

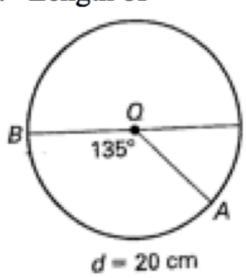
19. $\frac{10}{1} \cdot \frac{\pi}{3} = \frac{10\pi}{3} cm$



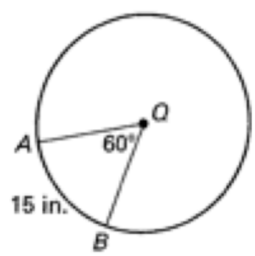
Example:

Find the indicated measure of each:

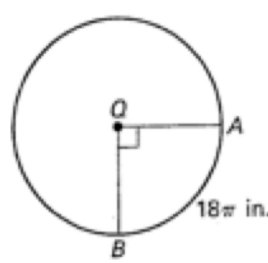
1. Length of $\widehat{AB}$



2. Circumference



3. Radius



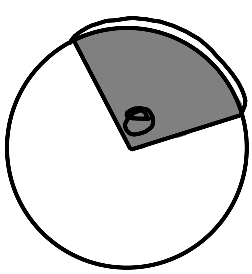
Recall:

Area of a circle

$$A = \pi r^2$$

Sectors:

A sector of a circle is the region (area) bounded by two radii of the circle and their intercept arc.



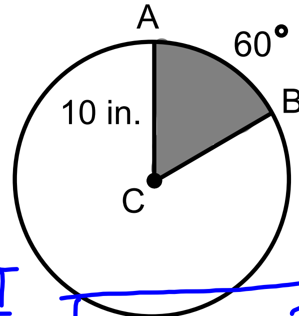
Degrees

$$Sector area = \left(\frac{\widehat{AB}}{360^o}\right)\underbrace{\pi r^2}_{\text{area}}$$

in², mi²

Example:

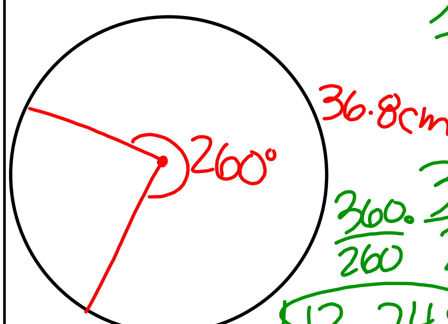
Find the area of the sector:



$$\frac{60°}{360°}\cdot\frac{\pi(10)^2}{1} = \frac{6000\pi}{360°} = \boxed{\frac{50\pi}{3}\ \text{in}^2}$$

23-25: arclength $= \left(\frac{\widehat{AB}}{360°}\right)\cdot\boxed{2\pi r}$ 26.

25. Find Radius



$$\frac{36.8}{2\pi} = \frac{260°}{360°}\cdot\frac{\cancel{2\pi} r}{\cancel{2\pi}}$$

$$\frac{360}{260}\cdot\frac{36.8}{2\pi} = \frac{\cancel{260}}{\cancel{360}}\cdot r\cdot\frac{\cancel{360}}{\cancel{260}}$$

$$\frac{13,248}{520\pi} = r$$

$$\boxed{\frac{1656}{65\pi}\text{cm}}\ r$$