

14-4 Solving Rational Equations

Book Section 9-3 pgs 498-504

Objectives:

I can solve a rational equation algebraically

I can identify extraneous solutions

I can solve real-world problems using rational equations

1 - excluded values \rightarrow denom. = 0
 $\frac{1}{x-2}$ $x \neq 2$

2 - Factor

3 - LCD

B $\frac{2x-9}{x-7} + \frac{x}{2} = \frac{5}{x-7}$

Identify any excluded values.

$x - 7 = 0$

$x = 7$

The excluded value is 7.

Identify the LCD.

The different factors are (x-7), 2.

The LCD is 2(x-7).

Multiply each term by the LCD.

$\frac{2x-9}{x-7} \cdot 2(x-7) + \frac{x}{2} \cdot 2(x-7) = \frac{5}{x-7} \cdot 2(x-7)$

Divide out common factors.

$\frac{2x-9}{\cancel{x-7}} \cdot 2 + \frac{x}{\cancel{2}} \cdot (x-7) = \frac{5}{\cancel{x-7}} \cdot 2$

Simplify.

$2(2x-9) + x(x-7) = 5(2)$

Use the Distributive Property.

$4x-18 + x^2-7x = 10$

Quad!

Write in standard form.

$x^2 - 3x - 28 = 0$

Factor.

$(x+4)(x-7) = 0$

Use the Zero Product Property.

$x+4=0$ or $x-7=0$

Solve for x.

$x = -4$ or $x = 7$

The solution $x = 7$ is extraneous because it is an excluded value. The only solution is $x = -4$.

$x = -4$

$\begin{array}{r} -28 \\ 1 \quad 28 \\ 2 \quad 14 \\ \hline +4 \quad -7 \end{array}$

Example 1 Solve each rational equation algebraically.

$$\textcircled{A} \quad \frac{3x+7}{x-5} = \frac{5x+17}{2(x-5)}$$

$$x \neq 5$$

$$\text{LCD: } (x-5)2$$

$$\frac{3x+7}{\cancel{x-5}} \cdot \cancel{2(x-5)} = \frac{5x+17}{\cancel{2(x-5)}} \cdot \cancel{2(x-5)}$$

$$2(3x+7) = 5x+17$$

$$\begin{array}{r} 6x + 14 = 5x + 17 \\ -5x \quad -14 \quad -5x \quad -14 \end{array}$$

$$x = 3$$

Your Turn

pg. 499

Solve each rational equation algebraically.

2. $\frac{8}{x+3} = \frac{x+1}{x+6}$

$x \neq -3, -6$

LCD: $(x+3)(x+6)$

$$\frac{8}{\cancel{x+3}} \cdot \cancel{(x+3)}(x+6) = \frac{\cancel{(x+1)}(x+3)\cancel{(x+6)}}{\cancel{x+6}}$$

$$8(x+6) = (x+1)(x+3)$$

$$\begin{array}{r} 8x+48 \\ -8x-48 \\ \hline \end{array} = \begin{array}{r} x^2+4x+3 \\ -8x-48 \\ \hline \end{array}$$

$$0 = x^2 - 4x - 45$$

$$0 = (x-9)(x+5)$$

$$x-9=0$$

$$x+5=0$$

$$\boxed{x=9 \quad x=-5}$$

$$\begin{array}{r} -45 \\ -9+5 \\ \hline \end{array} \quad -4$$

Solve:

$$\frac{2 \cdot \cancel{6x}}{\cancel{x} \cdot 6} - \frac{1 \cdot \cancel{6x}}{\cancel{6} \cdot 2x} = \frac{5 \cdot \cancel{6x}}{\cancel{2x} \cdot 3} - \frac{1 \cdot \cancel{6x}}{\cancel{6} \cdot 3}$$

$$x = 0$$

$$\text{LCD: } 6x$$

$$2(6) - 1(x) = 5(3) - 1(2x)$$

$$\frac{6x}{x} = 6 \quad \frac{6x}{6} = x \quad \frac{6x}{2x} = 3 \quad \frac{6x}{3} = 2x$$

$$\cancel{12} - x = 15 - \cancel{2x}$$

$$-12 + 2x - 12 + 2x$$

$$x = 3$$

Solve the rational equation algebraically

pg. 504

$$\frac{x^2 - 29}{(x-7)(x-3)} = \frac{6}{x-7} + \frac{5}{x-3}$$

$$x \neq 7, 3$$

$$LCD: (x-7)(x-3)$$

$$\frac{x^2 - 29}{\cancel{(x-7)(x-3)}} \cdot \cancel{(x-7)(x-3)} = \frac{6}{\cancel{x-7}} \cdot \cancel{(x-7)(x-3)} + \frac{5}{\cancel{x-3}} \cdot \cancel{(x-7)(x-3)}$$

$$x^2 - 29 = 6(x-3) + 5(x-7)$$

$$x^2 - 29 = 6x - 18 + 5x - 35$$

$$x^2 - 29 = 11x - 53$$

$$-1x + 53 - 11x + 53$$

$$x^2 - 11x + 24 = 0$$

$$(x-3)(x-8) = 0$$

$$x = \cancel{3} \text{ or } 8 \rightarrow \boxed{x=8}$$

10. Jake can mulch a garden in 30 minutes. Together, Jake and Ross can mulch the same garden in 16 minutes. How much time t , in minutes, will it take Ross to mulch the garden when working alone? pg. 506

$$\frac{1}{30} + \frac{1}{t} = \frac{1}{16}$$

$\begin{matrix} 2 \cdot 3 \cdot 5 \\ \times \end{matrix}$
 $\begin{matrix} t \\ \times \end{matrix}$
 $\begin{matrix} 2 \cdot 2 \cdot 2 \cdot 2 \\ \times \end{matrix}$

$$\text{LCD: } 2 \cdot 3 \cdot 5 \cdot t \cdot 2 \cdot 2$$

$$= 240t$$

$$\frac{1}{\cancel{30}} \cdot \cancel{240}t + \frac{1}{\cancel{t}} \cdot \cancel{240}t = \frac{1}{16} \cdot 240t$$

$$\cancel{8t} + 240 = 15t$$

$$- \cancel{8t} \quad \quad - 8t$$

$$\frac{240}{7} = \frac{7t}{7}$$

$$t = 34.3 \text{ min}$$

Ross

Your Turn

pg. 504

4. Kevin can clean a large aquarium tank in about 7 hours. When Kevin and Lara work together, they can clean the tank in 4 hours. Write and solve a rational equation to determine how long, to the nearest tenth of an hour, it would take Lara to clean the tank if she works by herself. Explain whether the answer is reasonable.

$$x=0$$

$$\text{LCD: } 7 \cdot t \cdot 4 = 28t$$

$$\frac{1}{7} \cdot 28t + \frac{1}{t} \cdot 28t = \frac{1}{4} \cdot 28t$$

$$\begin{array}{r} 3t + 28 = 7t \\ -3t \quad -3t \\ \hline 28 = 4t \\ \frac{4}{4} \quad \frac{4}{4} \\ \hline \boxed{t = 9.3} \end{array}$$