3.1 Zeroes of a Polynomial

Book Pages: 371-372

Objectives:

- I can find the zeroes of a polynomial by using the factor theorem, remainder theorem, and rational roots theorem

Goal: Factored Form

$x^2 - 7x + 12 = (x-3)(x-4)$

Standard

Factored

Linear X

Divide the following polynomials

$x+4\overline{)3x^2+7x-20}$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

$$\begin{array}{r|rrr} -4 & 3 & 7 & -20 \\ & \downarrow & -12 & 20 \\ \hline & 3 & -5 & ☺ \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & 7 & -3 & 1 \\ & \downarrow & 6 & 3 & 30 & 81 \\ \hline & 2 & 1 & 10 & 27 & \boxed{82} \end{array}$$

$3x^2+7x-20=(x+4)(3x-5)$

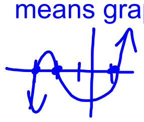
Standard ↓ ↓ factored

$2x^4-5x^3+7x^2-3x+1=(x-3)(2x^3+x^2+10x+27)+82$

↓

not factored!

Identify the zeros of the following and explain what that means graphically.



$f(x)=(x+2)(x-1)(x+3)=0$

$x+2=0 \quad x-1=0 \quad x+3=0$

$\quad\quad\quad +1\ +1 \quad -3\ -3$

zero's
x-int

$$\boxed{x=-2 \quad x=1 \quad x=-3}$$

Write the function in standard form and state the relationship between the degree and zeros of the function

$f(x)=(x+2)(x-1)(x+3)$

$(x^2+x-2)(x+3)$

$x^3+3x^2+x^2+3x-2x-6$

x^3+4x^2+x-6

deg: 3 zero's: 3 (x-int)

degree = zero's!

Remainder Theorem:

if remainder = 0, then x-a is a factor

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

magic box

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

$(x-a)(\quad)$

factor quotient also a factor

Remainder = ☺

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B $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5$; $(x + 1)$

Use synthetic division.

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & -6 & 4 & 5 \\ & & & & & \\ \hline & 1 & & & & \end{array}$$

Since the remainder is ____, $(x + 1)$ ____ a factor. Write $q(x)$.

$q(x) =$ ____

Now factor $q(x)$ by grouping.

$q(x) =$ ____

$=$ ____

$=$ ____

$=$ ____

So, $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5 =$ ____.

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Example 3 Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

(A) $p(x) = x^3 + 3x^2 - 4x - 12$; $(x + 3)$

Use synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ & & -3 & 0 & 12 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

Since the remainder is 0, $x + 3$ is a factor.

Write $q(x)$ and then factor it.

$q(x) = x^2 - 4 = (x + 2)(x - 2)$

So, $p(x) = x^3 + 3x^2 - 4x - 12 = (x + 2)(x - 2)(x + 3)$.

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Your Turn

1. Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$. 2.

8. ~~$p(x) = 2x^4 + 8x^3 + 2x + 8$; $(x + 4)$~~

$x^3 - x^2 - 10x - 8 : (x+1)$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & \downarrow & -1 & 2 & 8 \\ \hline \rightarrow & 1 & -2 & -8 & \boxed{0} \end{array}$$

x^2

$(x+1)(x^2 - 2x - 8)$ ← factor

$1 \cdot -8 = -8$

1 8

+2 −4

$\boxed{(x+1)(x+2)(x-4)}$

9. $p(x) = 3x^3 - 2x + 5$; $(x - 1)$

$$\begin{array}{r|rrrr} 1 & 3 & 0 & -2 & 5 \\ + & \downarrow & 3 & 3 & 1 \\ \hline & 3 & 3 & +1 & \boxed{6} \end{array}$$

$\cancel{0}$

$\boxed{\text{not a factor}}$

Fractions $\sqrt{2}$

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

Factor: 20

1, 2, 4, 5, 10, 20

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

factors of constant → term w/o variable

factors of leading coefficient → term w/ highest degree

not given a factor

Find the rational $\boxed{\text{zeros}}$ of the polynomial function; then write the function as a product of factors.

① x =

② ( )( )( )

$f(x) = 1x^3 + 2x^2 - 19x - 20$

$\pm \frac{1, 2, 4, 5, 10, 20}{1}$

① $x = -5, -1, 4$

② $f(x) = (x+5)(x+1)(x-4)$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$f(x) = 1x^4 - 4x^3 - 7x^2 + 22x + 24$

$\pm \frac{1, 2, 3, 4, 6, 8, 12, 24}{1}$

1 $x = -2, -1, 3, 4$

2 $f(x) = (x+2)(x+1)(x-3)(x-4)$

Find all the zeros $f(x) = x^3 - 2x^2 - 8x$

$\pm \frac{0}{1}$

$x + 0 = x$

$x - 0 = x$

1 $x = -2, 0, 4$

2 $f(x) = (x+2)(x)(x-4)$

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

Find all the zeros of: $f(x) = x^3 + x^2 - 14x + 6$

$\pm \frac{1, 2, 3, 6}{1}$ $x = 3$

3	1	1	-14	6
	↓	3	12	-6
	1	4	-2	0 ☺
	x^2	x	#	

-2

1 2

$(x-3)(x^2+4x-2) = 0$

$$\boxed{x = 3, \pm 2\sqrt{6}}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$f(x) = ax^2 + bx + c$$

x^2+4x-2

$a = 1$

$b = 4$

$c = -2$

$$x = \frac{-4 \pm \sqrt{(4)^2-4(1)(-2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16+8}}{2}$$

$$= \frac{-4 \pm \sqrt{24}}{2}$$

$\sqrt[2]{24}$

24 → 2, 12; 12 → 2, 6; 6 → 2, 3

$2\sqrt{6}$

$$= \frac{-4}{2} \pm \frac{2\sqrt{6}}{2}$$

$$\boxed{x = -2 \pm \sqrt{6}}$$

Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1 = x

$2(x+2)(x-4)(x-1)$

$2x^3$

3. $2x^3 - x^2 - 13x - 6$

$\pm \frac{1,2,3,6}{1,2} = \pm 1, 2, 3, 6, \frac{1}{2}, \frac{3}{2},$

$\boxed{x = -2, -\frac{1}{2}, 3}$

17. $f(x) = (x+3)(x-4)(x-3) = 0$

$x+3=0$

$x = -3, 4, 3$

deg zeros