

13. $x^3 - 4x^2 - 11x + 2$

$$\begin{array}{r} -2 \overline{) 1 \ -4 \ -11 \ 2} \\ \underline{ 2 \ 12 \ -2} \\ 1 \ -6 \ 1 \end{array} \quad \text{⊙}$$

$x^2 \quad x \quad \#$

$(x+2)(x^2 - 6x + 1)$

$a=1$
 $b=-6$
 $c=1$

$\sqrt{32}$
 \wedge
 $2 \ 16$
 $\text{⊙} \ 44$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)} = \frac{6 \pm \sqrt{32}}{2}$$

$$= \frac{-6 \pm \sqrt{32}}{2}$$

$$x = 3 \pm 2\sqrt{2}$$

$$\begin{aligned} x &= -2 \\ x &= 3 + 2\sqrt{2} \\ x &= 3 - 2\sqrt{2} \end{aligned}$$

14. $x^3 - 4x^2 + 2x + 4$

$$\begin{array}{r} 2 \overline{) 1 \ -4 \ 2 \ 4} \\ \underline{ 2 \ -4 \ -4} \\ 1 \ -2 \ -2 \end{array} \quad \text{⊙}$$

$x^2 \quad x \quad \#$

$(x-2)(x^2 - 2x - 2)$

$\begin{matrix} -2 \\ \wedge \\ 1 \ 2 \end{matrix}$

$$x = 2$$

$$x = \frac{2 \pm \sqrt{2^2 - 4(1)(-2)}}{2}$$

$$= \frac{2 \pm \sqrt{20}}{2}$$

$$= \frac{2 \pm 2\sqrt{5}}{2}$$

3-2 Graphing Polynomial Functions

(Book 5.4 pg. 293-306)

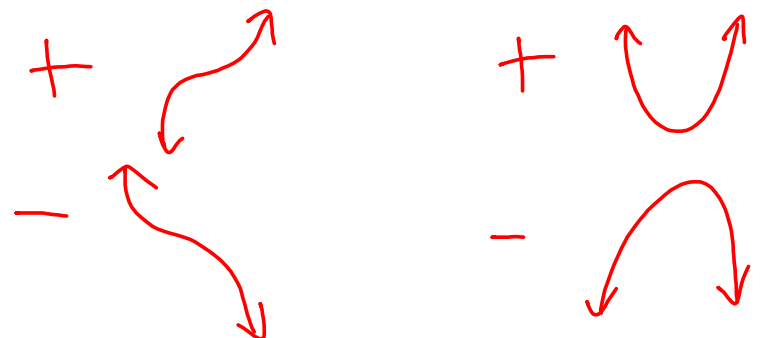
Objectives:

- I can graph a polynomial function by hand and using technology
- I can find end behavior of a polynomial function
- I can identify zeros, x-intercepts, and factors of a polynomial function
- I can determine the multiplicity of a polynomial function

Graphing Polynomials Task

$X^3 \rightarrow$ opposite

$X^4 \rightarrow$ same






$$f(x) = (x+3)(x-2)$$

$$x = -3, 2$$

End Behavior

Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

Function	Domain	Range	End Behavior
 $f(x) = x^2$	$(-\infty, \infty)$	$[0, \infty)$	As $x \rightarrow +\infty, f(x) \rightarrow \infty$ As $x \rightarrow -\infty, f(x) \rightarrow \infty$
 $f(x) = x^4$	$(-\infty, \infty)$	$(0, \infty)$	As $x \rightarrow +\infty, f(x) \rightarrow \infty$ As $x \rightarrow -\infty, f(x) \rightarrow \infty$
 $f(x) = x^6$	$(-\infty, \infty)$	$[0, \infty)$	As $x \rightarrow +\infty, f(x) \rightarrow \infty$ As $x \rightarrow -\infty, f(x) \rightarrow \infty$

Does it change if I have a negative coefficient? How?






D: $(-\infty, \infty)$ same
 P: $(-\infty, 0]$

As $x \rightarrow -\infty, f(x) \rightarrow -\infty$
 As $x \rightarrow \infty, f(x) \rightarrow -\infty$

End Behavior

Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

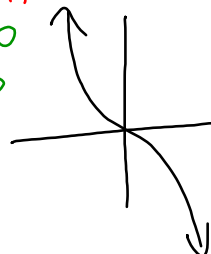
Function	Domain	Range	End Behavior
 $f(x) = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \rightarrow +\infty, f(x) \rightarrow \infty$ As $x \rightarrow -\infty, f(x) \rightarrow -\infty$
 $f(x) = x^3$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \rightarrow +\infty, f(x) \rightarrow \infty$ As $x \rightarrow -\infty, f(x) \rightarrow -\infty$
 $f(x) = x^5$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \rightarrow +\infty, f(x) \rightarrow \infty$ As $x \rightarrow -\infty, f(x) \rightarrow -\infty$

Does it change if I have a negative coefficient? How?

ODD Deg: End Behavior in opp. Directions

Pos LC: R $\rightarrow \infty$ L $\rightarrow -\infty$

Neg LC: R $\rightarrow -\infty$ L $\rightarrow \infty$



Zeros, x-intercepts, and factors

Find the factors of $f(x) = x^2 + 4x + 3$

$$(x+1)(x+3)$$

Now find the x-intercepts of $f(x) = x^2 + 4x + 3$

$$(-1, 0) \quad (-3, 0)$$

Lastly find the zeros of $f(x) = x^2 + 4x + 3$

$$x = -1, -3$$

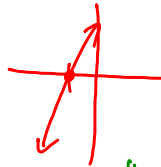
What is the same between the factors, x-intercepts, and zeros of this function?

Multiplicity $(x-1)^1(x+2)^2(x+1)^5$

The **power** of the factor determines the nature of the intersection at the point $x = a$.
(This is referred to as the **multiplicity**.)

Straight intersection:

$(x - a)^1$ The power of the zero is 1.



Tangent intersection : (bounce)

$(x - a)^{\text{even}}$ The power of the zero is even.

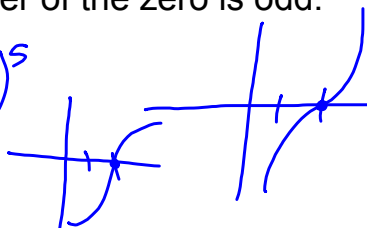
$$(x-1)^2 \text{ or } (x-1)^4$$



Inflection intersection: (like a slide through)

$(x - a)^{\text{odd}}$ The power of the zero is odd.

$$(x-2)^3 \quad (x-2)^5$$



- A Use a graphing calculator to graph the cubic functions $f(x) = x^3$, $f(x) = x^2(x - 2)$, and $f(x) = x(x - 2)(x + 2)$. Then use the graph of each function to answer the questions in the table.

Function	$f(x) = x^3$	$f(x) = x^2(x - 2)$	$f(x) = x(x - 2)(x + 2)$
How many distinct factors does $f(x)$ have?			
What are the graph's x -intercepts?			
Is the graph tangent to the x -axis or does it cross the x -axis at each x -intercept?			
How many turning points does the graph have?			
How many global maximum values? How many local?			
How many global minimum values? How many local?			

- B Use a graphing calculator to graph the quartic functions $f(x) = x^4$, $f(x) = x^3(x - 2)$, $f(x) = x^2(x - 2)(x + 2)$, and $f(x) = x(x - 2)(x + 2)(x + 3)$. Then use the graph of each function to answer the questions in the table.

(If time)

Function	$f(x) = x^4$	$f(x) = x^3(x - 2)$	$f(x) = x^2(x - 2)(x + 2)$	$f(x) = x(x - 2)(x + 2)(x + 3)$
How many distinct factors?				
What are the x -intercepts?				
Tangent to or cross the x -axis at x -intercepts?				
How many turning points?				
How many global maximum values? How many local?				
How many global minimum values? How many local?				

Reflect

- What determines how many x -intercepts the graph of a polynomial function in intercept form has?

- What determines whether the graph of a polynomial function in intercept form crosses the x -axis or is tangent to it at an x -intercept?

Graphing a Polynomial from factors

B $f(x) = -(x-4)(x-1)(x+1)(x+2)$

Identify the end behavior.

As $x \rightarrow +\infty$, $f(x) \rightarrow$

As $x \rightarrow -\infty$, $f(x) \rightarrow$

Identify the graph's x-intercepts, and then use the sign of $f(x)$ on intervals determined by the x-intercepts to find where the graph is above the x-axis and where it's below the x-axis.

The x-intercepts are $x =$, $x =$, $x =$, $x =$

Interval	Sign of the Constant Factor	Sign of $x-4$	Sign of $x-1$	Sign of $x+1$	Sign of $x+2$	Sign of $f(x) = -(x-4)(x-1)(x+1)(x+2)$
$x <$ <input type="text"/>	-		-		-	
<input type="text"/> $< x <$ <input type="text"/>	-		-		+	
<input type="text"/> $< x <$ <input type="text"/>	-		+		+	
<input type="text"/> $< x <$ <input type="text"/>	-		+		+	
$x >$ <input type="text"/>	-		+		+	

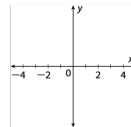
So, the graph of $f(x)$ is above the x-axis on the intervals

$< x <$ and $< x <$, and

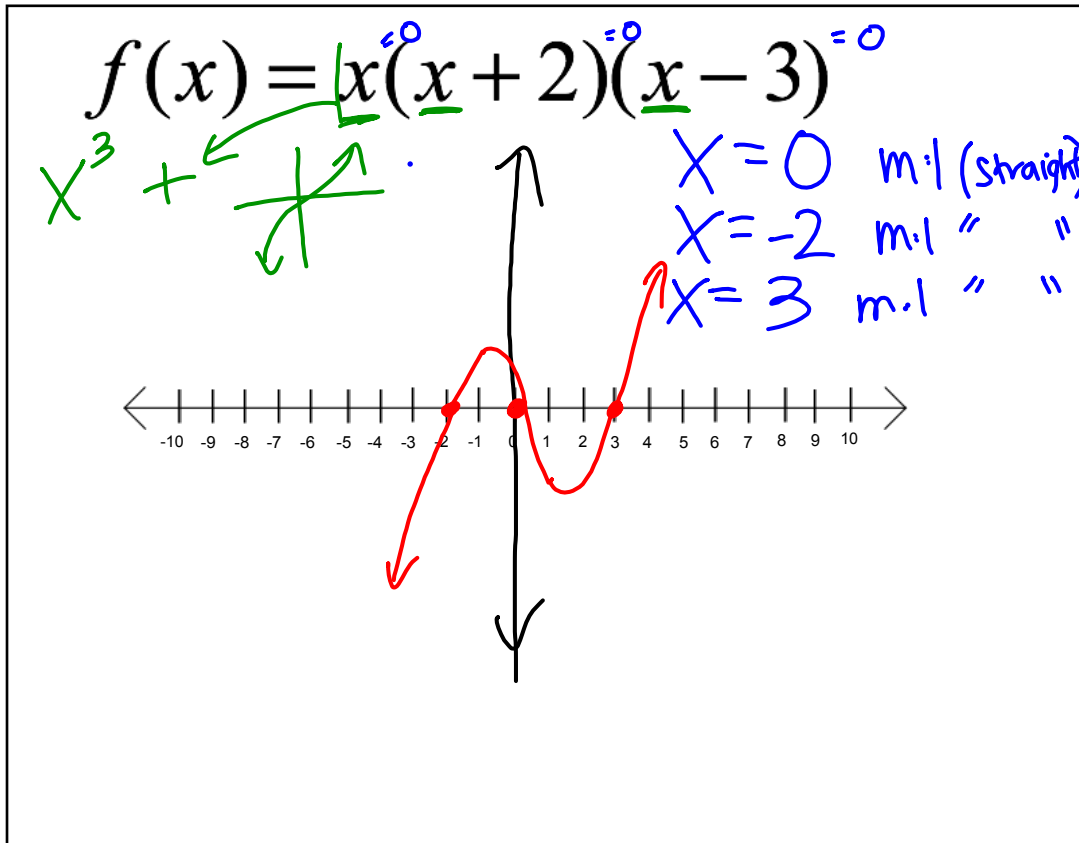
it's below the x-axis on the intervals $x <$, $< x <$,

and $x >$

Sketch the graph.



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$$f(x) = -\cancel{0}(x-4)(x-1)(x+1)(x+2)$$

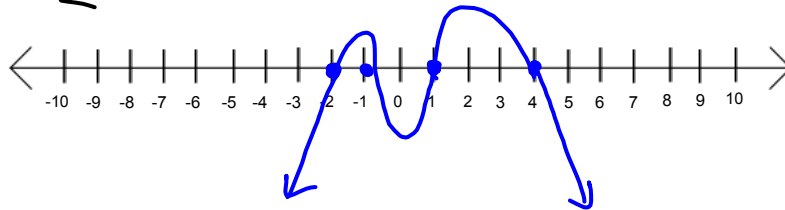
$$X = 4 \quad m:1$$

$$X = 1 \quad m:1$$

$$X = -1 \quad m:1$$

$$X = -2 \quad m:1$$

X^4 even



Ex. 8 Find the zeros, the multiplicity, end behavior and graph the following:

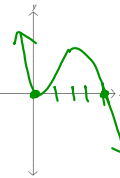
a. $f(x) = -x^2(x-4)$

$$-x^2(x-4)$$

$X = 0$ m:2 tangen/bounce

$X = 4$ m:1 straight

X^3 -



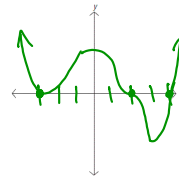
$$\begin{aligned} -x^2 &= 0 \\ \Rightarrow x &= 0 \\ \sqrt{x^2} &= 0 \\ x &= 0 \end{aligned}$$

b. $f(x) = (x+3)^2(x-2)^3(x-4)$

$X = -3$ m:2 bounce

$X = 2$ m:3 inflection

$X = 4$ m:1 straight



c. X^6 even +

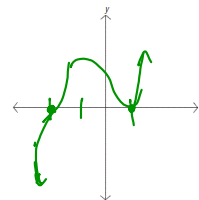
$$f(x) = (x+2)^3(x-1)^2$$

$$(x+2)^3(x-1)^2$$

$X = -2$ m:3 inflec.

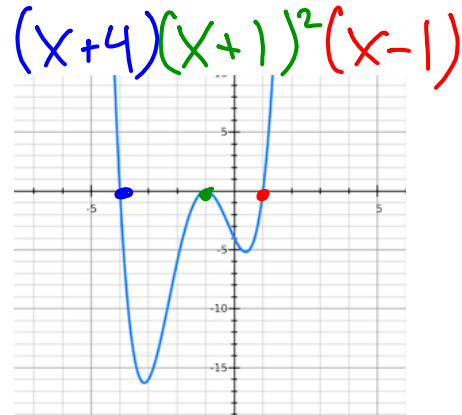
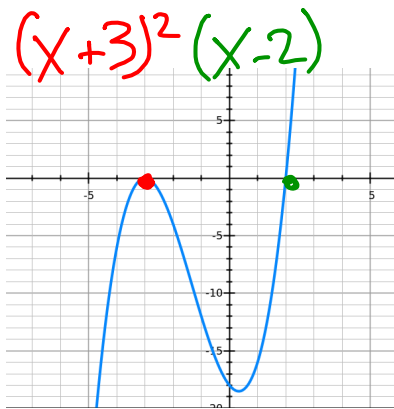
$X = 1$ m:2

X^9 +



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Write a function in intercept form for the given graphs whose intercepts are integers. Assume the constant factor of a is either 1 or -1.



Turning Point \rightarrow switch inc/dec

2 turning points

no t.p.

3 t.p.

global/absolute : WHOLE graph

local/relative : part of the graph

