4-1 Review of Complex Numbers

Objective: Students will be able to:

Know the parts of a complex number

Know how to add, subtract, and multiply 2 complex numbers

Know what a conjugate is and how to find one

$$(i)^2 = (\sqrt{-1})^2$$

or

$$i^2 = -1$$

Definition

Complex numbers are numbers of the form $a+bi$, where a and b are real numbers. The real number a is called the real part and the number b is called the imaginary part.

$a+bi$

r i

Identify the real and imaginary parts of each complex number.

$4+5i$

R: 4

I: 5

$5-i$

R: 5

I: -1

$3+0i$

R: 3

I: 0

$0+7i$

R: 0

I: 7

Write each of the following as a pure imaginary number.

$\sqrt{-16} = \sqrt{16 \cdot -1} = \sqrt{16}\sqrt{-1} = 4i$

$\sqrt{-3} = \sqrt{3}\sqrt{-1} = \sqrt{3}\,i$

$\sqrt{-18}$ $\sqrt{18}\sqrt{-1}$

$18 = 2 \cdot 9$

$9 = 3 \cdot 3$

$3\sqrt{2}\,i$

You Try

$\sqrt{-12}$

$2\sqrt{3}\,i$

$\sqrt{-5}$

$\sqrt{5}\,i$

$\sqrt{-36} = 6i$

Write each in Standard Form. State the real and imaginary parts.

$a+bi$

$2-\sqrt{-25}$

$2-5i$

R: 2

I: -5

$3+\sqrt{-50}$

$5 \cdot 10$

$5 \cdot 2$

$3+5\sqrt{2}\,i$

R: 3

I: $5\sqrt{2}$

$\frac{4-\sqrt{-12}}{2}$

$6 \cdot 2$

$3 \cdot 2$

$\frac{4-2\sqrt{3}\,i}{2} \Rightarrow \frac{4}{2} - \frac{2\sqrt{3}\,i}{2}$

$= 2-\sqrt{3}\,i$

You Try

$-2-\sqrt{-8}$

$-2-2\sqrt{2}\,i$

$r = -2$

$i = -2\sqrt{2}$

$\frac{6-\sqrt{-72}}{3}$

$36 \cdot 2$

$6 \cdot 6$

$\frac{6}{3} - \frac{6\sqrt{2}\,i}{3}$

$2-2\sqrt{2}\,i$

Add: like terms

$(4-3i)+(-2+5i)$

$2+2i$

$\left(4+\sqrt{-25}\right)+\left(-6-\sqrt{-16}\right)$

$(4+5i)+(-6-4i)$

$-2+i$

Subtract:

$(-3+7i)-(5-4i)$

$-3+7i-5+4i$

$-8+11i$

$\left(3+\sqrt{-12}\right)-\left(-2-\sqrt{-27}\right)$

$12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3$

$27 = 3 \cdot 9 = 3 \cdot 3 \cdot 3$

$(3+2\sqrt{3}i)-(-2-3\sqrt{3}i)$

$3+2 \quad 2\sqrt{3}i+3\sqrt{3}i$

$5+5\sqrt{3}i$

You Try

$\left(4-\sqrt{-4}\right)+\left(-7+\sqrt{-9}\right)$

$(4-2i)+(-7+3i)$ $3i$

$-3+i$

$-3+5i$

$(4-2i)-(-2+7i)$

$6-9i$

Multiply

$4i(3-6i)$

$12i-24i^2$

$12i-24(-1)$

$12i+24 \rightarrow 24+12i$

$4x(3-6x)$

$12x-24x^2$

$(-2+4i)(3-i)$

$-6+2i+12i=4i^2-1$

$+4$

$-2+14i$

Remember from before:

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$$

only works when $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers

$\sqrt{-4}\sqrt{-25} = \sqrt{-4 \cdot -25} = \sqrt{100}$

$= 10$ ~~$= 10$~~

$2i \cdot 5i = 10i^2 = -10$

This means that

$$\sqrt{a}\sqrt{b} \neq \sqrt{ab} \text{ if } a < 0 \text{ or } b < 0$$

$\sqrt{4}\sqrt{25} = \sqrt{4 \cdot 25} = \sqrt{100} = 10$

$2 \cdot 5 = 10$

Multiply

$$\sqrt{-25}\sqrt{-4}$$

$5i \cdot 2i = 10i^2 = -10$

$$\left(2+\sqrt{-16}\right)\left(1-\sqrt{-4}\right)$$

$(2+4i)(1-2i)$

$2 - 4i + 4i - 8i^2$

$+8$

10

You Try

$$\sqrt{-9}\sqrt{-36}$$

$$\left(2+\sqrt{-36}\right)\left(4-\sqrt{-25}\right)$$

Complex Conjugate

If $a+bi$ is a complex number, then its conjugate is defined as $a-bi$

$a-bi$ $a+bi$

Name the conjugate of the following complex numbers

$3+2i$ $\quad$ $4-3i$ $\quad$ $-16+32i$

$3-2i$ $\quad$ $4+3i$ $\quad$ $-16-32i$

$0-17i$ $\quad$ $+4i$

$0+17i$ $\quad$ $-4i$

$17i$

Multiply (What Happens?) → cancels out i's ☺

$(4+3i)(4-3i)$

$16-12i+12i-9i^2(-1)$

$+9$

25