# 8.1 Defining and evaluating logarithms

inverse exponential

$\sqrt{x^2}$ $\log_3 3^x$

$\frac{2x}{2}$ $\sqrt[2]{x} = x^{\frac{1}{2}}$

$x - 2$
$+2$

$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$\frac{1}{10^{-2}} = 10^2 = 100$

## Explain 1 Converting Between Exponential and Logarithmic Forms of Equations

In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = \log_b x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = \log_3 x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = \log_{\frac{1}{4}} x$. The inverse relationship between exponential functions and logarithmic functions also means that you can write any exponential equation as a logarithmic equation and any logarithmic equation as an exponential equation.

| Exponential Equation | Logarithmic Equation |
|---|---|
| $b^x = a$ | $\log_b a = x$ |

$$b > 0, b \neq 1$$

# Examples

Pg 791

| Exponential Equation | Logarithmic Equation |
|---|---|
| $4^3 = 64$ | $\log_4 64 = 3$ |
| $5^{-2} = \frac{1}{25}$ | $\log_5 \frac{1}{25} = -2$ |
| $\left(\frac{2}{3}\right)^p = q$ | $\log_{\frac{2}{3}} q = p$ |
| $\left(\frac{1}{2}\right)^n = m$ | $\log_{\frac{1}{2}} m = n$ |

P. 297

Switch between Log and exponential forms

| Exponential Equation | Logarithmic Equation |
|---|---|
| $3^5 = 243$ | $\log_3 243 = 5$ |
| $4^{-3} = \frac{1}{64}$ | $\log_4 \frac{1}{64} = -3$ |
| $\left(\frac{3}{4}\right)^r = s$ | $\log_{\frac{3}{4}} s = r$ |
| $\frac{1}{5}^w = v$ | $\log_{\frac{1}{5}} v = w$ |

$3^2 = 9$ $\log_3 9 = 2$

$3^x = 27$ $\log_3 27 = x$

The natural logarithm:

$y = \ln x$ is equivalent to $x = e^y$

ln

$\log_e$

The common logarithm:

$y = \log x$ is equivalent to $x = 10^y$

$\log_{10}$

(not in the book)

| Exponential Equation | Logarithmic Equation |
|---|---|
| $e^5 \approx 148.4$ | $\ln 148.4 = 5$ |
| $e^{1.8} \approx 6$ | $\ln 6 \approx 1.8$ |
| $10^5 = 100{,}000$ | $\log 100{,}000 = 5$ |
| $10^3 = 1000$ | $\log 1{,}000 = 3$ |

If $f(x) = \log_{10} x$, find $f(1000)$, $f(0.01)$, and $f(\sqrt{10})$.

P 793

$f(1000) = x$

$\log_{10} 1000 = x$

$10^x = 1000$

$10^x = 10^3$

$x = 3$

So, $f(1000) = 3$.

$f(0.01) = x$ P 795

$\log_{10} 0.01 = x$

$10^x = 0.01$

$10^x = 10^{-2}$

$x = -2$

So, $f(0.01) = -2$.

$f(\sqrt{10}) = x$

$\log_{10} \sqrt{10} = x$

$10^x = \sqrt{10}$

$10^x = 10^{\frac{1}{2}}$

$x = \frac{1}{2}$

So, $f(\sqrt{10}) = \frac{1}{2}$.

If $f(x) = \log_{\frac{1}{2}} x$, find $f(4)$, $f\left(\frac{1}{32}\right)$ and $f(2\sqrt{2})$.

P 793

$f(4) = x$

$\log_{\frac{1}{2}} 4 = x$

$\left(\frac{1}{2}\right)^x = 4$

$\left(\frac{1}{2}\right)^x = 2^{\square}$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{\square}$

$x = \square$

So, $f(4) = \square$.

$f\left(\frac{1}{32}\right) = x$

$\log_{\frac{1}{2}} \frac{1}{32} = x$

$\left(\frac{1}{2}\right)^x = \frac{1}{32}$

$\left(\frac{1}{2}\right)^x = \frac{1}{2^{\square}}$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{\square}$

$x = \square$

So, $f\left(\frac{1}{32}\right) = \square$.

$f(2\sqrt{2}) = x$

$\log_{\frac{1}{2}} 2\sqrt{2} = x$

$\left(\frac{1}{2}\right)^x = 2\sqrt{2}$

$\left(\frac{1}{2}\right)^x = \sqrt{2^2 \cdot 2}$

$\left(\frac{1}{2}\right)^x = \sqrt{2^{\square}}$

$\left(\frac{1}{2}\right)^x = 2^{\square}$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{\square}$

$x = \square$

So $f(2\sqrt{2}) = \square$.

**Your Turn**

P 794

**9.** If $f(x) = \log_7 x$, find $f(343)$, $f\left(\frac{1}{49}\right)$, and $f(\sqrt{7})$.

$y = \log_7 343$

$7^y = 343$

$7^y = 7^3$

$\boxed{y = 3}$

$y = \log_7 \frac{1}{49}$

$7^y = \frac{1}{49}$

$7^y = 7^{-2}$

$\boxed{y = -2}$

$y = \log_7 \sqrt{7}$

$7^y = \sqrt{7}$

$7^y = 7^{1/2}$

$y = 1/2$

Find the exact value without a calculator

(not in the book)

$\log_2 32 = y$

$2^y = 32$

$\boxed{y = 5}$

$\log_4 \frac{1}{16} = y$

$4^y = 1/16$

$\boxed{y = -2}$

$\log 10000000 = y$

$10^y = 10000000$

$\boxed{y = 7}$

$\log .00001 = y$

$10^y = .00001$

$\boxed{y = -5}$

You try

$\log_5 25 = y$

$5^y = 25$

$\boxed{y = 2}$

$\log_2 \frac{1}{8} = y$

$2^y = 1/8$ $\boxed{y = -3}$

$\log 1000$

$\log .001$

Use a calculator to

P 795

First, find the common logarithm of 0.42. Round the result to the thousandths place and raise 10 to that number to confirm that the power is close to 0.42.

$\log 0.42 \approx \square$

$10^{\square} \approx 0.42$

Next, find the natural logarithm of 0.42. Round the result to the thousandths place and raise $e$ to that number to confirm that the power is close to 0.42.

$\ln 0.42 \approx \square$

$e^{\square} \approx 0.42$

**Your Turn**

P 795

Use a scientific calculator to find the common logarithm and the natural logarithm of the given number. Verify each result by evaluating the appropriate exponential expression.

**11.** 0.25

$\log .25 = -.602$

$10^{-.602} = 0.25$

$\ln 0.25 = -1.386$

$e^{-1.386} = 0.25$

**12.** 4

$\log 4 = 0.602$

$\ln 4 = 1.386$

P 796

The acidity level, or pH, of a liquid is given by the formula $\boxed{pH = \log \frac{1}{[H^+]}}$ where $[H^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. In a typical chlorinated swimming pool, the concentration of hydrogen ions ranges from $\boxed{1.58 \times 10^{-8}}$ moles per liter to $\boxed{6.31 \times 10^{-8}}$ moles per liter. What is the range of the pH for a typical swimming pool?

$pH = \log\left(\frac{1}{(1.58 \times 10^{-8})}\right) = 7.8$

$pH = \log\left(\frac{1}{(6.31 \times 10^{-8})}\right) = 7.2$

$\boxed{7.2 - 7.8}$

The intensity level $L$ (in decibels, dB) of a sound is given by the formula $L = 10 \log \frac{I}{I_0}$ where $I$ is the intensity (in watts per square meter, W/m$^2$) of the sound and $I_0$ is the intensity of the softest audible sound, about $10^{-12}$ W/m$^2$. What is the intensity level of a rock concert if the sound has an intensity of 3.2 W/m$^2$?

P 797

$$L = 10 \cdot \log\left(\frac{3.2}{10^{-12}}\right) = \boxed{125.1\ \text{dB}}$$