$f(x) = \log_3 x$ $\qquad \sqrt{27}$

$y = \log_3 \sqrt{27}$

$3^y = \sqrt{27} \rightarrow 3^y = 27^{\frac{1}{2}}$

$3^y = 3^{\frac{3}{1} \cdot \frac{1}{2}}$

$3^y = 3^{\frac{3}{2}}$

$\boxed{y = 3/2}$

$y = \log_6 6\sqrt[3]{6}$

$6^y = 6\sqrt[3]{6}$

$6^y = \sqrt[3]{6^4}$

$6^y = 6^{\frac{4}{3}}$

$\boxed{y = 4/3}$

$\sqrt[3]{8}$

8

2 4

2 2

2

25. a. $\ln e^2 = y$

$$e^y = e^2$$

$$\boxed{y = 2}$$

b. $10^{\log 7} = y$

$$\log_{10} y = \log_{10} 7$$

$$\boxed{y = 7}$$

16. 19

$$\log 19 = 1.279$$

$$\ln 19 = 2.944$$

36. $10 \cdot \log\left(\frac{\cancel{\text{intensity}}\ 10^{-10}}{10^{-12}}\right) = 20\ \text{dB}$

8-2 Properties of Logarithms

$$\log_a 1 = 0 \qquad \log_a a = 1$$

Evaluate

$\log_8 1 = y$

$8^y = 1$

$\boxed{y=0}$

$\log_7 7 = y$

$7^y = 7$

$\boxed{y=1}$

$\log_5 1$

$\boxed{0}$

$\ln 1$

$\boxed{0}$

$\log_4 4$

$\boxed{1}$

$\log_{10} 10$

$\boxed{1}$

Inverse Property of Logarithms

If b and M are positive real numbers, with $b \neq 0$, then

$$\cancel{b}^{\log_b M} = M$$

Evaluate

$$\cancel{5}^{\log_5 20}$$

$$\boxed{20}$$

$$\cancel{8}^{\log_8 \sqrt{23}}$$

$$\sqrt{23}$$

You Try

$$12^{\log_{12} \sqrt{2}}$$

$$\sqrt{2}$$

$$10^{\log 0.2}$$

$$0.2$$

Inverse Property of Logarithms

If b and r are positive real numbers, with $b \neq 0$, then

$$\cancel{\log_a a}^{r} = r$$

exp

Evaluate

$$\cancel{\log_4 4}^{3}$$

$$\boxed{3}$$

$$\ln e^{-0.5}$$

e

$$\boxed{-0.5}$$

You Try

$$\log_8 8^{1.2}$$

$$\boxed{1.2}$$

$$\log 10^{-4}$$

$$\boxed{-4}$$

Product Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b(MN) = \log_b M + \log_b N$$

mult → +

Write each of the following logarithms as the sum of logarithms.

$\log_2(5 \cdot 3)$

$\log_2 5 + \log_2 3$

$\ln(6z)$

$\ln 6 + \ln z$

You Try

$\log_4(9 \cdot 5)$

$\log_4 9 + \log_4 5$

$\log(5w)$

$\log_{10} 5 + \log_{10} w$

Quotient Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$\div \longrightarrow -$

$$\log_2\left(\frac{5}{3}\right)$$

$\log_2 5 - \log_2 3$

$$\log\left(\frac{y}{5}\right)$$

$\log y - \log 5$

You Try

$$\log_7\left(\frac{9}{5}\right)$$

$\log_7 9 - \log_7 5$

$$\ln\left(\frac{p}{3}\right)$$

$\ln p - \ln 3$

Write the following as the sum or difference of logarithms.

$$\log_3\left(\frac{4x}{y}\right)$$

$\log_3 4 \cdot x - \log_3 y$

$(\log_3 4 + \log_3 x) - \log_3 y$

You Try

$$\log_3\left(\frac{3m}{n}\right)$$

$\log_3 3 \cdot m - \log_3 n$

$\cancel{\log_3 3^1} + \log_3 m - \log_3 n$

$(1 + \log_3 m) - \log_3 n$

$$\log_3\left(\frac{q}{3p}\right)$$

$\log_3 q - (\log_3 3p)$

$\log_3 q - (\log_3 3 + \log_3 p)$

$\log_3 q - (1 + \log_3 p)$

Power Rule of Logarithms
If M and b are positive real numbers, with $b \neq 0$, then

$$\log_b M^r = r\log_b M$$

Use the power Rule of Logarithms to express all powers as factors.

$\log_8 3^5$

$5\log_8 3$

$\ln x^{\sqrt{3}}$

$\sqrt{3}\ln x$

You Try

$\log_2 5^{1.6}$

$1.6\log_2 5$

$\log b^5$

$5\log b$

Expand the logarithm.

$\log_2(x^2 y^3)$

$\log_2 x^2 + \log_2 y^3$

$2\log_2 x + 3\log_2 y$

100

10 -

$\log\left(\frac{100x}{\sqrt{y}}\right)$

$\log 100x - \log\sqrt{y}$

$(\log 100 + \log x) - \log\sqrt{y}$

$(\log_{10} 10^2 + \log x) - \log y^{1/2}$

$\boxed{2 + \log x - \frac{1}{2}\log y}$

You Try

$\sqrt[3]{n} = n^{\frac{1}{3}}$

$\log_4(a^2 b)$ $\log_4 a^2 + \log_4 b$

$2\log_4 a + \log_4 b$

$\log_3\left(\frac{9m^4}{\sqrt[3]{n}}\right)$ $\log_3 9 \cdot m^4 - \log_3 \sqrt[3]{n}$

$(\log_3 9 + \log_3 m^4) - \log_3 \sqrt[3]{n}$

$\cancel{\log_3 3^2} + 4\log_3 m - \log_3 n^{1/3}$

$\boxed{2 + 4\log_3 m - \frac{1}{3}\log_3 n}$

Write each of the following as a single logarithm.

$\log_6 3 + \log_6 12$

$\log_6(3\cdot 12)$

$\log_6 36$

$\cancel{\log_6} 6^2 = \boxed{2}$

$\log(x-2) - \log x$

$\boxed{\log \frac{x-2}{x}}$

$\frac{\cancel{x}}{\cancel{x}} - \frac{2}{x}$

$1 - \frac{2}{x}$

You try

$\log_8 4 + \log_8 16$

$\log_8(4\cdot 16)$

$\log_8 64 \rightarrow$

$\cancel{\log_8} 8^2 = \boxed{2}$

$\log_3(x+4) - \log_3(x-1)$

$\log_3 \frac{x+4}{x-1}$

Write each of the following as a single logarithm.

$$2\log_2(x-1)+\frac{1}{2}\log_2 x$$

$\log_2(x-1)^2 + \log_2 x^{\frac{1}{2}} \rightarrow \sqrt{x}$

$\log_2\left[(x-1)^2\sqrt{x}\right]$

$$\log(x-1)+\log(x+1)-3\log x \xrightarrow{} 3$$

$\log\left[(x-1)(x+1)\right] - \log x^3$

$\log \dfrac{(x-1)(x+1)}{x^3}$

You Try

$$\log_5 x - 3\log_5 2$$

$$\log_2(x+1)+\log_2(x+2)-2\log_2 x^2$$

$\log_2 (x+1)(x+2) - \log_2 x^2$

$\log_2 \dfrac{(x+1)(x+2)}{x^2}$

Rewrite and express in terms of a and b given that a=ln3 and b=ln4

ln12

HW

ln16

Change of Base Formula

If $a \neq 0$, $b \neq 0$, and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

which means:

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

Use your calculator to approximate the following:

$$\log_4 45 = 2.75$$

$$\frac{\log 45}{\log 4} = 2.75$$

$$\frac{\ln 45}{\ln 4} = 2.75$$

Summary of Properties

$$\cancel{\log_a a}^r = r \qquad \cancel{b}^{\cancel{\log_b} M} = M$$

$$\log_b(M \cdot N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b M^r = r \log_b M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$

21. $a = \ln 2$ $\quad b = \ln 3$

$\ln 9$

$\ln(3 \cdot 3) = \ln 3 + \ln 3$

$b + b$

$\boxed{2b}$

23. $\ln 6$ $\quad \sqrt{\ln 2}$

$\ln(3 \cdot 2) = \ln 3 + \ln 2$

$\boxed{b + a}$

$a + b$

$\ln 2^{1/2}$

$\boxed{\frac{1}{2}a}$