

5.  $8m^2 + 4m - 16 = -m^2$   
 $+m^2$   $+m^2$

$$9m^2 + 4m - 16 = 0$$

$a = 9$   $ax^2 + bx + c = 0$   
 $b = 4$   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   
 $c = -16$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(9)(-16)}}{2(9)}$$

$$x = \frac{-4 \pm \sqrt{16 + 576}}{18}$$

$\sqrt{592}$

$$x = \frac{-4 \pm \sqrt{592}}{18}$$

$2 \cdot 296$   
 $2 \cdot 148$   
 $2 \cdot 74$   
 $2 \cdot 37$   
 $2 \cdot 2\sqrt{37}$   
 $4\sqrt{37}$

$$x = \frac{-4 \pm 4\sqrt{37}}{18}$$

$$x = \frac{-4}{18} \pm \frac{4\sqrt{37}}{18}$$

$$x = \frac{-2}{9} \pm \frac{2\sqrt{37}}{9}$$

$$x = \frac{-2 \pm 2\sqrt{37}}{9}$$

7.  $4b^2 + 8b + 7 = 4$   
 $-4 -4$   
 $4b^2 + 8b + 3 = 0$

$a = 4$   
 $b = 8$   
 $c = 3$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{-8 \pm \sqrt{64 - 48}}{8}$$

$$x = \frac{-8 \pm \sqrt{16}}{8}$$

$\frac{8+4}{8} = \frac{12}{8} = \frac{3}{2}$   
 $\frac{8-4}{8} = \frac{4}{8} = \frac{1}{2}$

## 8-2 Complex Operations

Objective: Students will be able to:

Know the parts of a complex number

Know how to add, subtract, and multiply 2 complex numbers

Reduce different powers of  $i$

imaginary

$$i = \sqrt{-1}$$

$$(i)^2 = (\sqrt{-1})^2$$

$$i^2 = -1$$

or

$$i^2 = -1$$

## Definition

Complex numbers are numbers of the form  $a+bi$ , where  $a$  and  $b$  are real numbers. The real number  $a$  is called the real part and the number  $b$  is called the imaginary part.

## Complex number

Standard form

$$\begin{array}{c} \underline{a} + \underline{bi} \\ \downarrow \quad \downarrow \\ \text{real} \quad \text{imaginary} \end{array}$$

Identify the real and imaginary parts of each complex number.

$$4 + 5i$$

$$\begin{array}{l} R: 4 \\ I: 5 \end{array}$$

$$5 - 1i$$

$$\begin{array}{l} R: 5 \\ I: -1 \end{array}$$

$$3 + 0i$$

$$\begin{array}{l} R: 3 \\ I: 0 \end{array}$$

$$0 + 7i$$

$$\begin{array}{l} R: 0 \\ I: 7 \end{array}$$

Write each of the following as a pure imaginary number.

$$\begin{aligned}\sqrt{-16} &= \sqrt{16 \cdot -1} \\ &= \sqrt{16} \cdot \sqrt{-1} \\ &= \boxed{4i}\end{aligned}$$

$$\begin{aligned}\sqrt{-3} &= \sqrt{3 \cdot -1} \\ &= \sqrt{3} \cdot \sqrt{-1} \\ &= \sqrt{3}i\end{aligned}$$

$$\begin{aligned}\sqrt{-18} &= \sqrt{18 \cdot -1} \\ &= \sqrt{18} \cdot \sqrt{-1} \\ &= \sqrt{3 \cdot 3 \cdot 2} \cdot \sqrt{-1} \\ &= 3\sqrt{2}i\end{aligned}$$

You Try

$$\begin{aligned}\sqrt{-12} &= \sqrt{12} \cdot \sqrt{-1} \\ &= \sqrt{3 \cdot 4} \cdot \sqrt{-1} \\ &= 2\sqrt{3} \cdot i = \boxed{2\sqrt{3}i}\end{aligned}$$

$$\begin{aligned}\sqrt{-5} &= \sqrt{5 \cdot -1} \\ &= \sqrt{5} \cdot \sqrt{-1} \\ &= \sqrt{5}i\end{aligned}$$

$$\begin{aligned}\sqrt{-36} &= \sqrt{36} \cdot \sqrt{-1} \\ &= 6 \cdot i = \boxed{6i}\end{aligned}$$

Write each in Standard Form. State the real and imaginary parts.

$a+bi$   
 $2 - \sqrt{-25}$   
 $2 - \sqrt{25}i$   
 $2 - 5i$   
 R: 2  
 I: -5

$3 + \sqrt{-50}$   
 $3 + \sqrt{50}i$   
 $3 + 5\sqrt{2}i$   
 R: 3  
 I:  $5\sqrt{2}$

$\frac{4 - \sqrt{-12}}{2}$   
 $\frac{4 - 2\sqrt{3}i}{2}$   
 $\frac{4}{2} - \frac{2\sqrt{3}i}{2}$   
 $2 - \sqrt{3}i$   
 R: 2  
 I:  $-\sqrt{3}$

You Try

$-2 - \sqrt{-8}$   
 $-2 - 2\sqrt{2}i$   
 $-2 - 2\sqrt{2}i$   
 R: -2  
 I:  $-2\sqrt{2}$

$\frac{6 - \sqrt{-72}}{3}$   
 $\frac{6 - 6\sqrt{2}i}{3}$   
 $2 - 2\sqrt{2}i$   
 R: 2  
 I:  $-2\sqrt{2}$

Add:

$$(4 - 3i) + (-2 + 5i)$$

$$\underline{4} - \underline{3i} - \underline{2} + \underline{5i}$$

$$\boxed{2 + 2i}$$

$$\frac{4 + -2 + -3i + 5i}{\boxed{2 + 2i}}$$

$$(4 + \sqrt{-25}) + (-6 - \sqrt{-16})$$

$$(4 + 5i) + (-6 - 4i)$$

$$4 + -6 + 5i + -4i$$

$$\boxed{-2 + i}$$

Subtract:

$$(-3 + 7i) - (5 - 4i)$$

$$\underline{-3} + \underline{7i} - \underline{5} + \underline{4i}$$

$$\boxed{-8 + 11i}$$

$$(3 + \sqrt{-12}) - (-2 - \sqrt{-27}) \quad \sqrt{27} \sqrt{-1}$$

$$(3 + \sqrt{12} i) - (-2 - \sqrt{27} i)$$

$$\begin{array}{c} \swarrow 2 \quad \searrow 6 \\ 2 \quad 3 \\ \swarrow \quad \searrow \end{array}$$

$$\begin{array}{c} \swarrow 3 \quad \searrow 9 \\ 3 \quad 3 \\ \swarrow \quad \searrow \end{array}$$

$$(3 + 2\sqrt{3} i) - (-2 - 3\sqrt{3} i)$$

$$3 + +2 + 2\sqrt{3}i + +3\sqrt{3}i$$

$$\boxed{5 + 5\sqrt{3}i}$$

You Try

$$(4 - \sqrt{-4}) + (-7 + \sqrt{-9})$$

$$(4 - 2i) - (-2 + 7i)$$

Multiply

$$4i(3 - 6i)$$

$$12i - 24i^2$$

$$12i - 24(-1)$$

$$12i + 24$$

$$\boxed{24 + 12i}$$

$$(-2 + 4i)(3 - i)$$

$$i^2 = \sqrt{-1}^2$$

$$i = -1$$

a + bi

$$17. (2+i)(4+3i)$$

$$8 + 6i + 4i + 3i^2$$

$$8 + 6i + 4i + 3(-1)$$

$$\underline{8} + \underline{6i} + \underline{4i} - \underline{3}$$

$$\boxed{5 + 10i}$$

$$19. (3+2i)^2 = (3+2i)(3+2i)$$



Remember from before:

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$$

only works when  $\sqrt[n]{a}$  and  $\sqrt[n]{b}$  are real numbers

This means that

$$\sqrt{a}\sqrt{b} \neq \sqrt{ab} \text{ if } a < 0 \text{ or } b < 0$$

Multiply

$$\sqrt{-25}\sqrt{-4}$$

$$(2 + \sqrt{-16})(1 - \sqrt{-4})$$

You Try

$$\sqrt{-9}\sqrt{-36}$$

$$(2 + \sqrt{-36})(4 - \sqrt{-25})$$

Evaluate

$$i^1$$

$$i^5$$

$$i^2$$

$$i^6$$

$$i^3$$

$$i^7$$

$$i^4$$

$$i^8$$

Evaluate:

$$i^{34}$$

$$i^{101}$$

You Try

$$i^{43}$$

$$i^{98}$$

The rest is Honors

Multiply (What Happens?)

$$(4 + 3i)(4 - 3i)$$

Complex Conjugate

If  $a+bi$  is a complex number, then its conjugate is defined as  $a-bi$

Divide:

$$\frac{3 + 4i}{2i}$$

$$\frac{-3 + i}{5 + 3i}$$

You Try

$$\frac{-4 + i}{3i}$$

$$\frac{4 + 3i}{1 - 3i}$$