8-4 Fundamental Theorem of Algebra

Objective: SWBAT discover and understand the Fundamental Theorem of Algebra and use quadratics in contextual applications.

Language Objective: Students can communicate the meaning of the Fundamental Theorem of Algebra by identifying possible root combinations for different polynomials.

Fundamental Theorem of Algebra Task

	Polynomial $f(x) = x^n + \ldots$	Degree	Factor	# of roots	Draw Graph and circle the x-intercept(s).
1	$f(x) = x^2 - 5x + 6$	2	$(x-2)(x-3)$ $x=2$ $x=3$	2	
2	$f(x) = x^2 - x - 2$	2	$(x+1)(x-2)$ $x=-1$ $x=2$	2	
3	$f(x) = x^2 - 2x - 15$	2	$(x-5)(x+3)$ $x=5$ $x=-3$	2	
4	$f(x) = x^3 + 4x^2 - 21x$	3	$x(x+7)(x-3)$ $x=0,-7,3$	3	
5	$f(x) = x^3 + 4x^2 + 3x$	3	$x(x+3)(x+1)$ $x=0,-3,-1$	3	
6	$f(x) = x^3 + 9x^2 + 18x$				
7	$f(x) = x^4 - 9x^2 + 18$		$\pm\sqrt{3}$ $\pm\sqrt{6}$ $(x^2-3)(x^2-6)$	4	

1. $x-2=0$ $x-3=0$
$+2\ +2$ $+3\ +3$
$x=2$ $x=3$

2. $x+1=0$ $x-2=0$
$-1\ -1$ $+2\ +2$
$x=-1$ $x=2$

4. $x=0$ $x+7=0$ $x-3=0$
$x=-7$ $x=3$

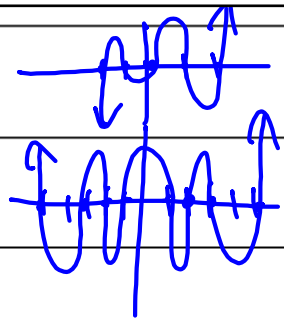
7. $x^2-3=0$ $x^2-6=0$
$+3\ +3$ $+6\ +6$
$\sqrt{x^2}=\sqrt{3}$ $\sqrt{x^2}=\sqrt{6}$
$x=\pm\sqrt{3}$ $x=\pm\sqrt{6}$
$\approx \pm 1.73$ $\approx \pm 2.44$

Factor $1x^2 - 1x - 2$ $\quad 1 \cdot -2 = -2$, $+1 -2$

$x^2 + x - 2x - 2$

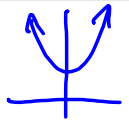
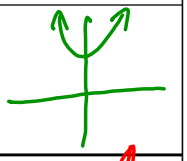
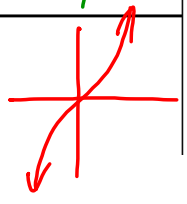
$x(x+1) - 2(x+1)$

$(x+1)(x-2)$

8	$x^5 - 5x^3 + 4x$	5	$x(x+2)(x+1)(x-1)(x-2)$ $x = 0, -2, -1, 1, 2$	5	
9	$x^8 - 25x^6 + 209x^4 - 126x^2 + 900$	8	$(x-3)(x+3)(x-2)(x+2)$ $(x+1)(x-1)(x+5)(x-5)$ $3\ -3\ 2\ -2$ $-1\ 1\ -5\ 5$	8	

Fill in the blank

10. Number of x-intercepts = Number of roots = degree

	Polynomial (nth degree) $f(x) = x^n + \ldots$	Degree	Find all roots, real and imaginary	# of real roots	# of imaginary roots	Draw Graph and circle the x-intercept(s).
11	$f(x) = x^2 + 25$	2	$x = \frac{0 \pm \sqrt{0^2 - 4(1)(25)}}{2(1)}$ $x = \pm 5i$	0	2	
12	$f(x) = 4x^2 + x + 9$	2	$x = \frac{-1 \pm \sqrt{143}i}{8}$	0	2	
13	$x^2 + 9 = 0$ $-9 \quad -9$ $\sqrt{x^2} = \sqrt{-9}$ $x = \pm 3i$	2	$x = \pm 3i$	0	2	
14	$f(x) = 5x^5 + 36x^3 + 7x$	5	$x(x^2 + 7)(5x^2 + 1)$ $x = 0$ $x = \pm\sqrt{7}i$ $x = \pm\sqrt{\frac{1}{5}}i$	0	4	

$x = 0 \qquad x^2 + 7 = 0 \qquad 5x^2 + 1 = 0$

15	$f(x) = x^6 - 2x^4 - 4x^2 + 8$		$(x^2 + 2)(x^2 - 2)^2$			
16	$f(x) = x^8 - 26x^4 + 25$	8	$(x^2 + 5)(x^2 - 5)(x^2 + 1)(x - 1)(x + 1)$ $x = \pm\sqrt{5}i$ 2I $x = \pm\sqrt{5}$ 2R $x = \pm i$ 2I $x = 1$ 1R $x = -1$ 1R	4	4	

17. Fill in the blank

The Fundamental Theorem of Algebra tells us that: The number of solutions = the number of real roots + the number of imaginary roots = degree and complex roots come in PAIRS!

$x^2 + 5 = 0 \qquad x^2 - 5 = 0 \qquad x^2 + 1 = 0$

$x - 1 = 0 \qquad x + 1 = 0$

a) $x^4 - 3x^3 + 6x^2 + 2x - 60$

9. 4R, 0C or 2R, 2C or 0R, 4C

10. 4R, 0C or 2R, 2C

11. 4R, 0C or 2R, 2C

12. 4R, 0C

b) $x^5 + 12$ 5 → 5 roots

9b) 5R, 0I
3R, 2I
1R, 4I

10, 11, 12
10, 11, 12 Imaginaries PAIRS
10

Check for understanding:

State the number of complex zeros and the possible combinations of the number of real and imaginary zeros for each function.

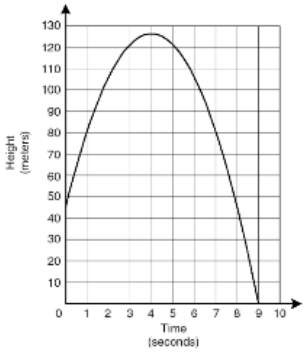
$f(x) = x^2 + 6x - 38$ $\quad f(x) = x^4 - 9x^2 + 18$

$f(x) = 5x^5 + 36x^3 + 7x$ $\quad f(x) = 3x^3 + 2x^2 - x$

Contextual Applications of Quadratics

1. A rocket was shot upward with an initial velocity of 144 feet per second. The height of the rocket is a function of t, the time in seconds since the rocket left the ground. The height can be expressed by the equation **$h(t) = 144t - 16t^2$.** How many seconds will it take for the rocket to return to the ground?

2. The graph below shows the height of a baseball from the time it is thrown from the top of a building to the time it hits the ground.



A. How much time elapses while the baseball is 80 meters or more above the ground?

B. When does the baseball reach its highest point?

C. How long has the ball been in the air when it hits the ground?

D. About how tall is the building from which the ball is thrown?

E. How can we tell from the graph that the ball was thrown and not just dropped?

A publishing company can get 1,000 subscribers for a new magazine if the monthly subscription rate is $5. It will get 100 more subscribers for each $.10 decrease in the monthly rate. What monthly rate will produce the maximum monthly income, and what is that income?

A rectangular dog kennel is connected to the side of my house. It is enclosed by 60 ft of fencing on three sides and on the fourth side is my house. Find the maximum area that can be enclosed in this way. What are the dimensions of the dog kennel?