11. $x \approx 1.022$

$x \approx 0.341$

10. $\ln(x-3) + \ln(x+4) = 3\ln 2 \quad \ln 2^3$

$e^{\ln(x-3)(x+4)} = e^{\ln 8}$

$(x-3)(x+4) = 8$

Review: $x^2 + 4x - 3x - 12 = 8$

$-8 \qquad -8$

$= 0$

$a = 1 \qquad 1x^2 + 1x - 20 = 0 \qquad -20 \qquad 1$

$(x-4)(x+5) = 0$

$1 \quad 20$

$2 \quad 10$

$-4 \quad 5$

$\boxed{x = 4, -5}$

7. $\cancel{\log_5} \boxed{5^{\frac{x}{4}}} = \log_5 30$

$$4 \cdot \frac{x}{4} = \boxed{\log_5 30} \cdot 4$$

$$= \left(\frac{\log 30}{\log 5}\right) \cdot 4$$

8-4 Graphing Logarithmic Functions

Book 15.2

Objectives:

1. I can identify the transformations performed on a logarithmic function.

2. I can graph a logarithmic function by hand.

3. I can identify the asymptote of a logarithmic function.

Logarithms & Exponentials

$f(x) = 2^x$ & $f(x) = \log_2 x$ are inverses

$y = 2^x$

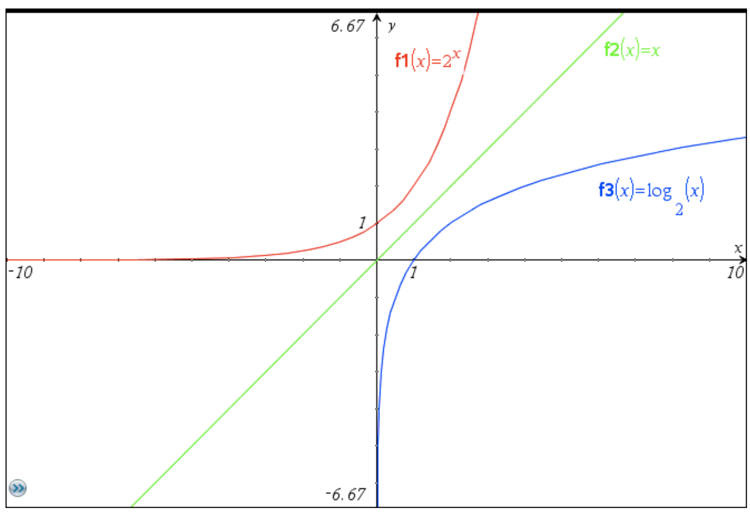
$x = 2^y$

$y = \log_2 x$

to find inverse:

1. switch x&y
2. solve for y

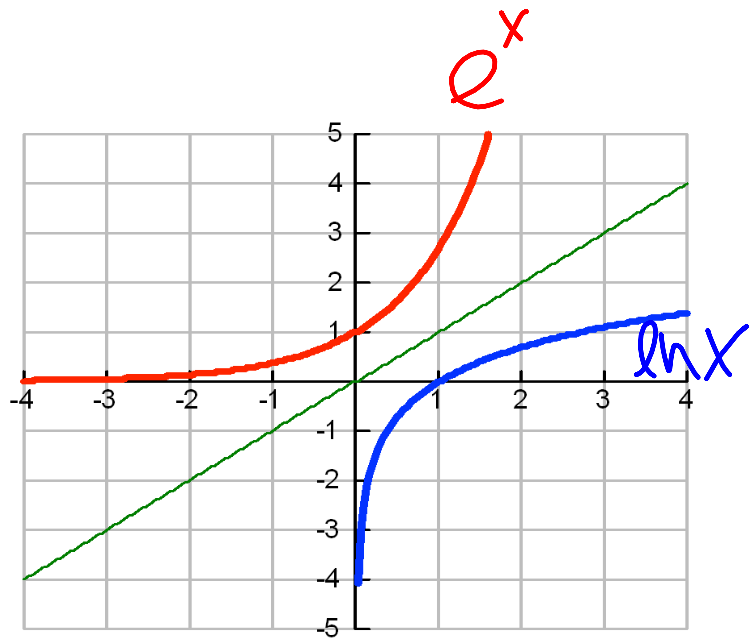
y=x



natural log

$$f(x) = \ln x$$

$$f(x) = e^x$$



Complete the table for the function $f(x) = \log x$

Then plot the points on the graph and connect the dots.

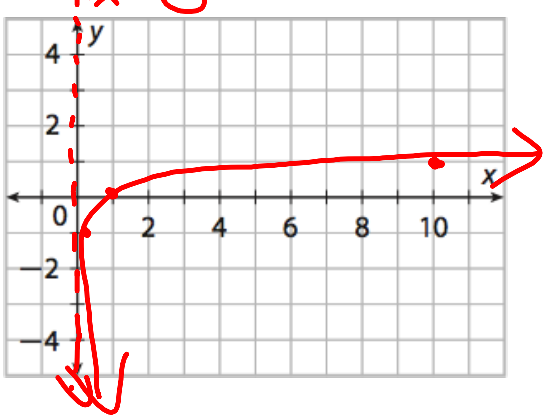
x	$f(x) = \log x$
0.1	-1
1	0
10	1

$y = \log 0.1 \rightarrow 10^y = \frac{1}{10}$

$y = \log 1 \rightarrow 10^y = 1$

$y = \log 10 \rightarrow 10^y = 10$

$x = 0$



Complete the table for the function $f(x) = \ln x$

Then plot the points on the graph and connect the dots.

x	$f(x) = \ln x$
$\frac{1}{e} \approx 0.368$	-1
1	0
$e \approx 2.72$	1
$e^2 \approx 7.39$	2

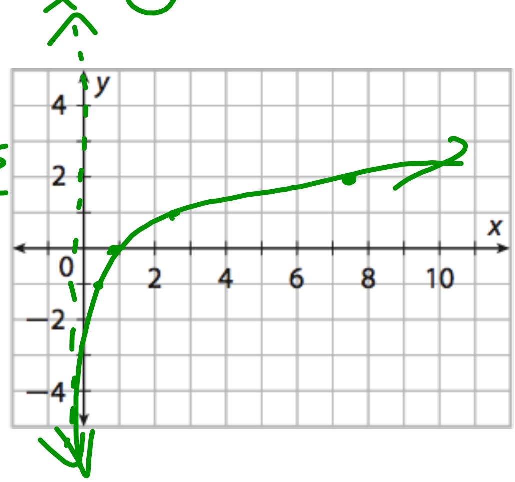
$\ln \frac{1}{e} \to e^y = \frac{1}{e}$

$e^y = 1$

$e^y = e$

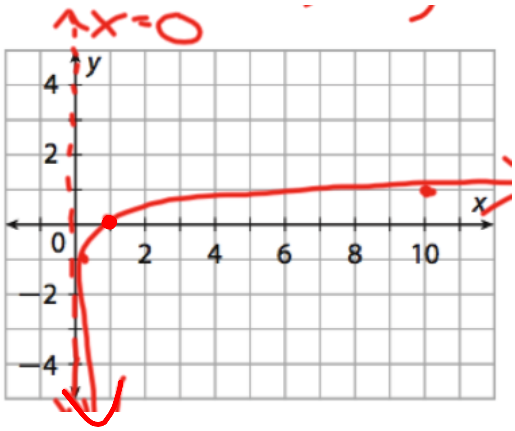
$e^y = e^2$

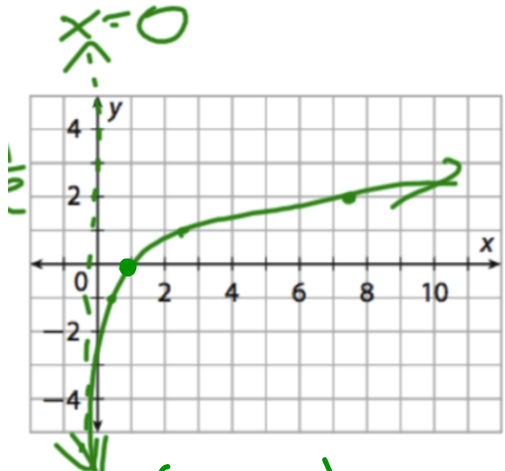
$x = 0$



Analyze the graphs of:

$f(x) = \log x$ $\qquad\qquad$ $f(x) = \ln x$





	$f(x) = \log x$	$f(x) = \ln x$
Domain:	$(0, \infty)$	$(0, \infty)$
Range:	$(-\infty, \infty)$	$(-\infty, \infty)$
End behavior:	left: $\lim_{x \to 0^+} f(x) = -\infty$ right: $\lim_{x \to \infty} f(x) = \infty$	$\lim_{x \to 0^+} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$
VA/HA:	VA: $x = 0$	VA: $x = 0$
Increasing/ Decreasing: x values	inc: $(0, \infty)$	inc: $(0, \infty)$
Intercepts: (x, y)	$(1, 0)$	$(1, 0)$

$$-a\log_b(x-h)+k$$

y's are wise :)

ref. x-axis

V.S.

R or L x's lie

U or D

Describe the transformations on each graph:

$$f(x) = \log(x+2)$$

left 2

$$f(x) = 3\log(x) - 4$$

V.S. down 4

$$f(x) = -2\ln(\cancel{2}x) + 5$$

ref. x-axis V.S. up 5

$$f(x) = 3\log_5(x-3) + 1$$

V.S. 3 right 3 up 1

Graphing Transformed Logarithmic Functions

When graphing a transformed function, it is helpful to consider the following features of the graph: the vertical asymptote, and two reference points (1,0) and (b,1).

Function	$f(x) = \log_b x$	$g(x) = a \log_b (x - h) + k$
Asymptote	$x = 0$	$x = h$
Reference point	$\boxed{(1, 0)}$	$(1 + h, k)$
Reference point	$\boxed{(b, 1)}$	$(b + h, a + k)$

asymptote x-slie

base of log

(B) $g(x) = 2\log(x+2) + 4$

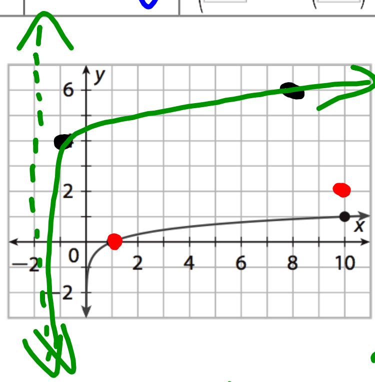
The transformations of the graph of $f(x) = \log x$ that produce the graph of $g(x)$ are as follows:

- a vertical stretch by a factor of 2
- a translation of 2 units to the left and 4 units up

Note that the translation of 2 units to the left affects only the x-coordinates of points on the graph of $f(x)$, while the vertical stretch by a factor of 2 and the translation of 4 units up affect only the y-coordinates.

Function	$f(x) = \log x$	$g(x) = 2\log(x+2) + 4$
Asymptote	$x = 0$	$x = \square$
Reference point	$(1, 0)$	$(\square - 2, 2(\square) + 4) = (\square, \square)$
Reference point	$(10, 1)$	$(\square - 2, 2(\square) + 4) = (\square, \square)$

$2\log(x+2)+4$



left 2
up 4

x	y
1	$0 \cdot 2 = 0$
10	$1 \cdot 2 = 2$

$x = -2$

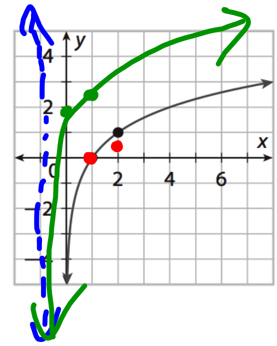
Domain: $\{x \mid x > \square\}$

Range: $\{y \mid -\infty < y < \square\}$

Your Turn

Identify the transformations of the graph of $f(x) = \log_b x$ that produce the graph of the given function $g(x)$. Then graph $g(x)$ on the same coordinate plane as the graph of $f(x)$ by applying the transformations to the asymptote $x = 0$ and to the reference points $(1, 0)$ and $(b, 1)$. Also state the domain and range of $g(x)$ using set notation.

2. $g(x) = \frac{1}{2}\log_2(x + 1) + 2$



$g(x) = \frac{1}{2}\log_2\left(\boxed{x+1}\right) + 2$

x	y	
1	0	$\cdot\frac{1}{2} = 0$
2	1	$\cdot\frac{1}{2} = \frac{1}{2}$

up 2

left 1

$x = -1$

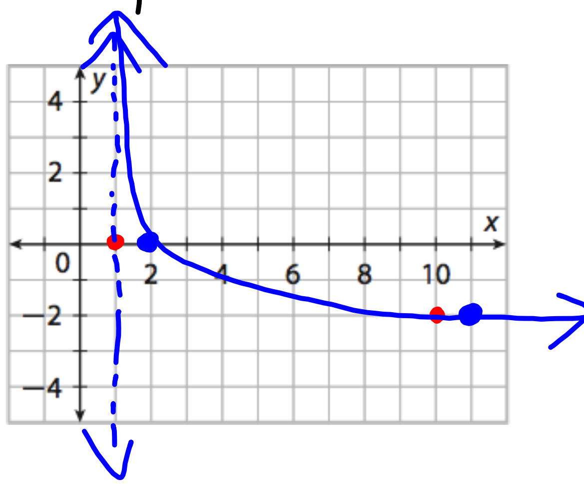
Graph and analyze the following functions:

$$f(x) = -2 \cdot \log(x-1)$$

right 1

$x = 1$

x	y
1	$0 \cdot -2 = 0$
10	$1 \cdot -2 = -2$



Domain:

Range:

End behavior:

VA/HA:

Increasing/ Decreasing:

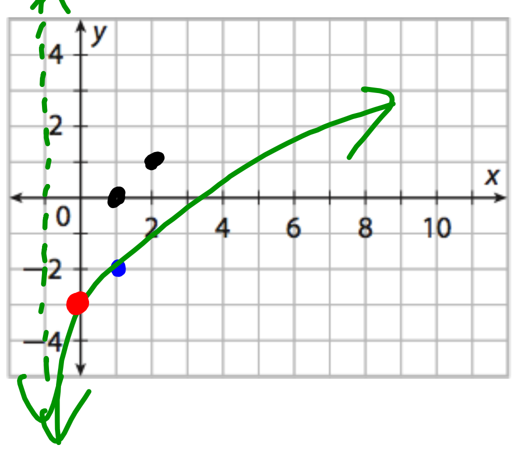
Intercepts:

$f(x) = \log_2(\underline{x+1}) - 3$ left 1 down 3

x	y
1	0
2	1

Domain: $(-1, \infty)$

x=-1



Range: $(-\infty, \infty)$

End behavior: left $\lim_{x \to -1^+} f(x) = -\infty$

right $\lim_{x \to \infty} f(x) = \infty$

VA/HA: VA: $x = -1$

Increasing/Decreasing: inc: $(-1, \infty)$ → domain

Intercepts: $(0, -3)$

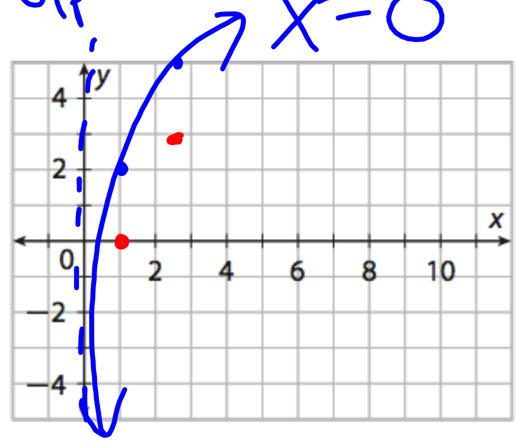
↳ key pts

$f(x) = 3 \cdot \ln_e(x) + 2$ $e \approx 2.7$

x	y
1	$0 \cdot 3 = 0$
2.7	$1 \cdot 3 = 3$

up 2

$x = 0$



Domain:

Range:

End behavior:

VA/HA:

Increasing/ Decreasing:

Intercepts:

3\. a-f describe transformations
domain & range

5-8: graph
domain & range