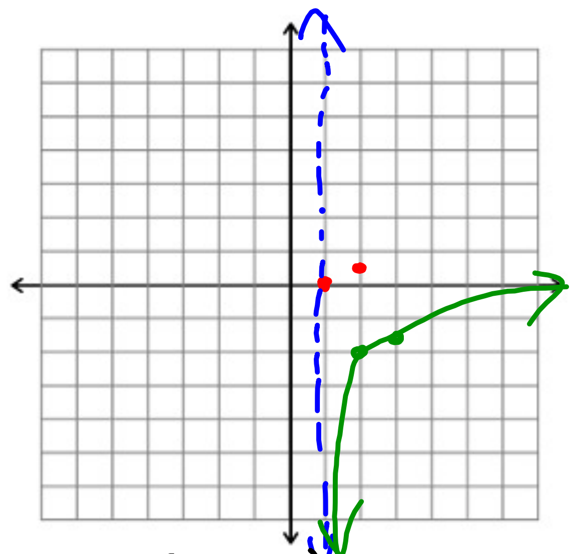
6. $\frac{1}{2}\log_2(x-1)-2$

right 1
down 2

x	y
1	$0 \cdot \frac{1}{2} = 0$
2	$1 \cdot \frac{1}{2} = \frac{1}{2}$

$x=1$



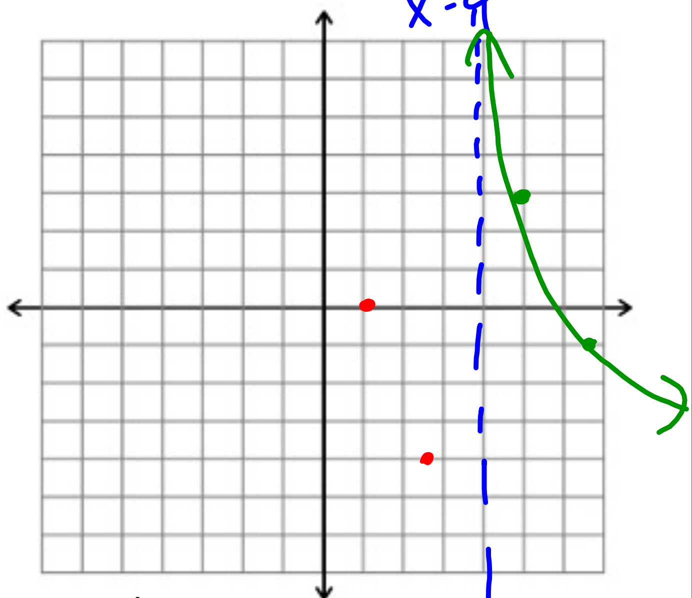
D: $(1,\infty)$
R: $(-\infty,\infty)$

7. $-4\ln(x-4)+3$

right 4
up 3

x	y
1	$0 \cdot -4 = 0$
e ≈ 2.7	$1 \cdot -4 = -4$

$x=4$



D: $(4, \infty)$
R: $(-\infty, \infty)$

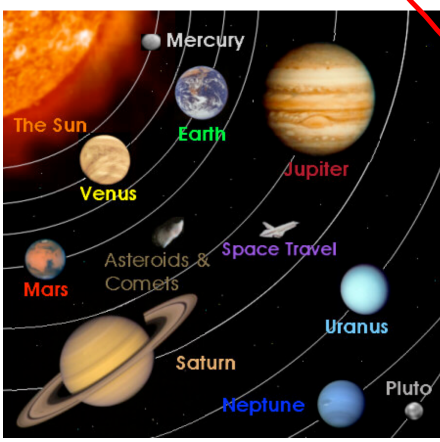
8.5 Modeling with Logarithms

Objectives:

I can use logarithms to solve real world problems.

I can solve interest equations by hand using logarithms.

Orders of Magnitude:



Mercury is 57.9 billion meters from the sun whereas Pluto is 5900 billion meters from the Sun.

a) Write the distance away from the sun for each planet in scientific notation.

b) How many times farther is pluto away from the sun than Mercury?

c) take the common log of Mercury's distance and pluto's distance and compare the difference.

The common logarithm of a positive quantity is its order of magnitude

Allows us to compare sizes that have a wide range between them:

i.e.: Pluto's distance from the Sun is 2 orders of magnitude greater than Mercury's

A kilometer is 3 orders of magnitude longer than a meter

A dollar is 2 orders of magnitude greater than a penny

Logarithmic Scales are used in many important applications in your (yes, your) life.

Decibel Scale - Sound

Richter Scale - earthquakes

Octave Scale -

F-Scale in Photography

pH Scale - acitity

Brightness of Stars -

Ban and Deciban -

Palermo Technical Impact Hazard Scale

Decibel Scale

Source	Intensity	Intensity Level	# of Times Greater Than TOH
Threshold of Hearing (TOH)	$1*10^{-12}$ W/m^2	0 dB	10^0
Rustling Leaves	$1*10^{-11}$ W/m^2	10 dB	10^1
Whisper	$1*10^{-10}$ W/m^2	20 dB	10^2
Normal Conversation	$1*10^{-6}$ W/m^2	60 dB	10^6 1000000
Busy Street Traffic	$1*10^{-5}$ W/m^2	70 dB	10^7
Vacuum Cleaner	$1*10^{-4}$ W/m^2	80 dB	10^8
Large Orchestra	$6.3*10^{-3}$ W/m^2	98 dB	$10^{9.8}$
Walkman at Maximum Level	$1*10^{-2}$ W/m^2	100 dB	10^{10}
Front Rows of Rock Concert	$1*10^{-1}$ W/m^2	110 dB	10^{11}
Threshold of Pain	$1*10^{1}$ W/m^2	130 dB	10^{13}
Military Jet Takeoff	$1*10^{2}$ W/m^2	140 dB	10^{14}
Instant Perforation of Eardrum	$1*10^{4}$ W/m^2	160 dB	10^{16}

Richter Scale

Magnitude	Description	Earthquake effects	Frequency of occurrence
Less than 2.0	Micro	Micro earthquakes, not felt.[13]	Continual
2.0–2.9	Minor	Generally not felt, but recorded.	1,300,000 per year (est.)
3.0–3.9		Often felt, but rarely causes damage.	130,000 per year (est.)
4.0–4.9	Light	Noticeable shaking of indoor items, rattling noises. Significant damage unlikely.	13,000 per year (est.)
5.0–5.9	Moderate	Can cause major damage to poorly constructed buildings over small regions. At most slight damage to well-designed buildings.	1,319 per year
6.0–6.9	Strong	Can be destructive in areas up to about 160 kilometres (99 mi) across in populated areas.	134 per year
7.0–7.9	Major	Can cause serious damage over larger areas.	15 per year
8.0–8.9	Great	Can cause serious damage in areas several hundred kilometres across.	1 per year
9.0–9.9		Devastating in areas several thousand kilometres across.	1 per 10 years (est.)
10.0+	Massive	Never recorded, widespread devastation across very large areas; see below for equivalent seismic energy yield.	Extremely rare (Unknown/May not be possible)

Comparing Earthquake intensities:

On the Richter scale, the magnitude M of an earthquake depends on the amount of energy, E (measured in ergs), released by the earthquake as follows:

magnitude

$$\boxed{M = \frac{2}{3}\log\frac{E}{10^{11.8}}}$$

energy - ergs

~~How many times more severe is a 7.4 quake than a 5.5 quake?~~ Find the magnitude of an earthquake that released 8.9×10^{21} ergs.

$$M = \frac{2}{3}\log\left(\frac{(8.9\times10^{21})}{10^{11.8}}\right) = \boxed{6.77}$$

The 1978 Mexico city earthquake had a magnitude level of 7.9 = M
What was the energy level? ______ ergs

$$M = \frac{2}{3}\log\frac{\boxed{E}}{10^{11.8}}$$

$$\frac{3}{2}\cdot 7.9 = \cancel{\frac{3}{2}}\cdot\cancel{\frac{2}{3}}\log\left(\frac{E}{10^{11.8}}\right)$$

$$11.85 = \log_{10}\frac{E}{10^{11.8}}$$

$$10 \qquad \cancel{10}$$

$$10^{11.8}\cdot 10^{11.85} = \frac{E}{\cancel{10^{11.8}}}\cancel{10^{11.8}}$$

$$10^{11.8}\cdot 10^{11.85} = E$$

$$4.47\times 10^{23} \text{ ergs}$$

10.91756103E19

10.92×10^{19}

$$\frac{a^5}{a^3} = \frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot a}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = a^2$$

$$a^{5-3} = a^2$$

$$\frac{x^7}{x^9} = x^{-2}$$

Comparing acidity: $pH = -\log\left[H^{+}\right]$

H^{+} hydrogen-ion concentration

Sour Vinegar has a pH of 2.4 and a box of Leg and Sickle baking soda has a pH of 8.4

a) what are their hydrogen-ion concentrations $[H^{+}]$

b) how many times greater is the H+ of vinegar than baking soda?

~~c) By how many order of magnitude do they differ?~~

Vinegar:

$2.4 = -\log\boxed{[H^{+}]}$

$\frac{2.4}{-1} = \frac{-\log[H^{+}]}{-1}$

$-2.4 = \log_{10}[H^{+}]$

$10 \quad 10$

$\boxed{10^{-2.4} = [H^{+}]}$

Baking Soda

$8.4 = -\log\boxed{[H^{+}]}$

$\frac{8.4}{-1} = \frac{-\log[H^{+}]}{-1}$

$-8.4 = \log[H^{+}]$

$10 \quad \cancel{10}$

$\boxed{10^{-8.4} = [H^{+}]}$

$\frac{\text{vin}}{\text{B.S.}} = \frac{10^{-2.4}}{10^{-8.4}} = 10^{-2.4+8.4} = \boxed{10^{6}}$

$= 1{,}000{,}000$

Carbonated water has a pH of 3.9 and household ammonia has a pH of 11.9.

a) what are their hydrogen-ion concentrations

b) how many times greater is the H+ of water than ammonia?

~~c) By how many order of magnitude do they differ?~~

C.W. $10^{-3.9}$ A. $10^{-11.9}$

$$\frac{10^{-3.9}}{10^{-11.9}} = \boxed{10^{8}}$$

Newton's Law of Cooling

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

This law states that the temperature difference between an object (T) and its surroundings (T_s) decreases exponentially as a function of time (t). Where T_0 is the initial temperature of the object, and $-k$ is our constant of variation representing the constant rate of decrease in the temperature difference.

A cup of cocoa has cooled from 95^o to 50^o after 13 minutes in a room at 25^o. How long will it take for the cup to cool to 30^o?

TIME = MONEY

Compounded annually:

$$A = P(1+r)^t$$

Ex. 1 Eric invests $500 at 7% interest compounded annually. Find the value of his investment 10 years later.

$$A = 500(1+.07)^{10}$$

$$= \boxed{\$983.57}$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$n \leftarrow$ # year/compounded

Ex. 3 Roger has \$500 to invest at 9% annual interest rate compounded monthly. How long will it take for his investment to grow to \$3000?

time

$$3000 = 500\boxed{\left(1+\frac{.09}{12}\right)^{12t}}$$

$$\frac{3000}{500} = \frac{500\boxed{(1.0075)^{12t}}}{500}$$

$$\log_{1.0075} 6 = \cancel{\log_{1.0075}} 1.0075^{12t}$$

#

$$\frac{\boxed{\log_{1.0075} 6}}{12} = \frac{\cancel{12}t}{\cancel{12}}$$

$$\left(\frac{\log 6}{\log 1.0075}\right) \div 12 = \boxed{19.98 \text{ yrs}}$$

end amount   initial

$$\boxed{A = Pe^{rt}}$$

Ex. 6 Mrs. McClelland saving account has a 1% interest rate compounded continuously. If she has $2000 in her savings account, how long will it take her to make $500 in interest?

$$2500 = 2000e^{.01t}$$