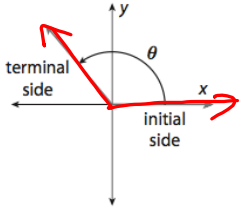
9-1 Angles and Radians Review

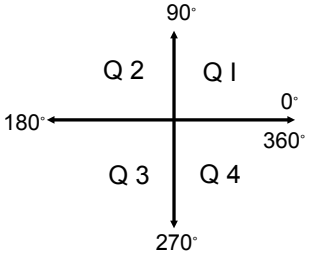
Book 18-1

Objectives:

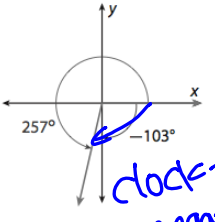
I can find co-terminal and reference angles

I can convert from radians to degrees and vice versa

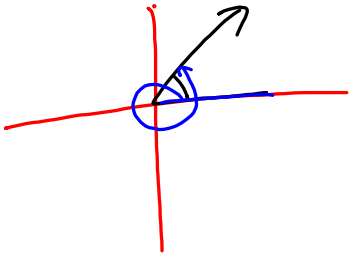




- If the rotation for an angle θ is less than 1 revolution in a counterclockwise direction, then the measure of θ is between 0° and 360°. An angle of rotation measured *clockwise* from standard position has a *negative* angle measure. **Coterminal angles** are angles that share the same terminal side. For example, the angles with measures of 257° and −103° are coterminal, as shown.



clock-wise negative

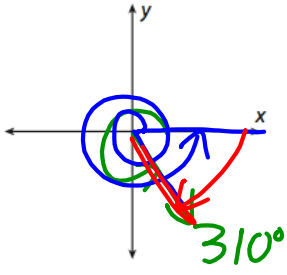


Pos = θ + 360° (angle)

neg = θ − 360°

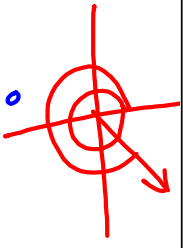
(θ: theta)

(A) Draw an angle of rotation of 310°. In what quadrant is the terminal side of the angle?



(B) On the same graph from the previous step, draw a positive coterminal angle. What is the angle measure of your angle?

360° + 310° = 670°

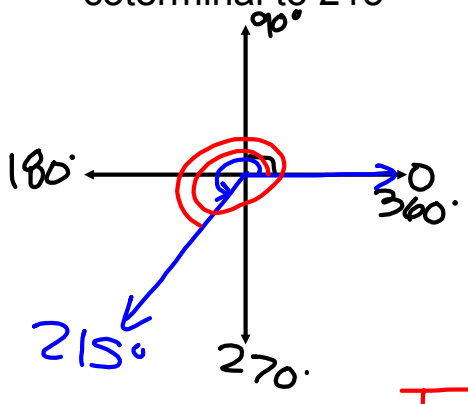


(C) On the same graph from the previous two steps, draw a negative coterminal angle. What is the angle measure of your angle?

310° − 360° = −50°

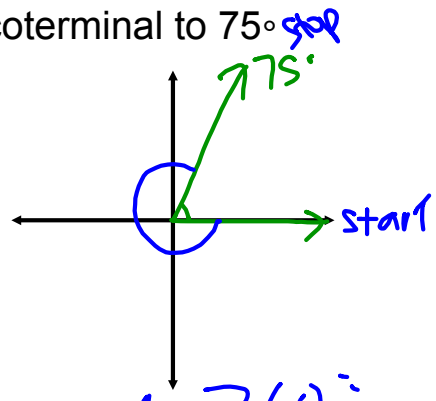
You Try! Draw and give the measure of the new angle

A positive angle coterminal to 215°



$215° + 360° = \boxed{575°}$

A negative angle coterminal to 75°



$75° - 360° = \boxed{-285°}$

For each angle, find the nearest two positive coterminal angles and the nearest two negative coterminal angles.

-102°

$-102° + 360° = \boxed{258°}$

$-102° - 360° = \boxed{-462°}$

328°

$328° + 360° = \boxed{688°}$

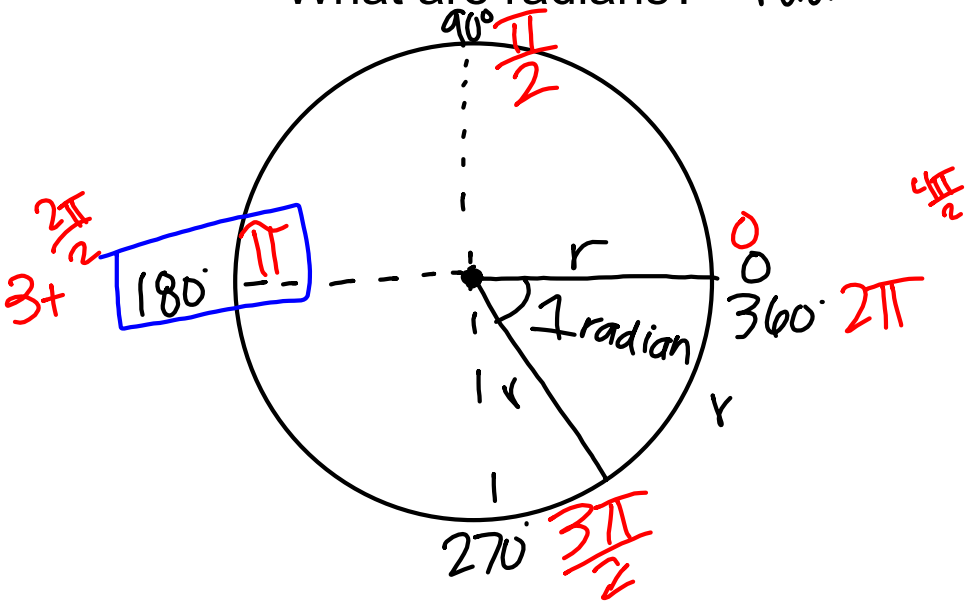
$328° - 360° = \boxed{-32°}$

You Try:

19° $379°$ $-341°$

225°

What are radians? radius



radians → fraction

degrees → decimal

CONVERTING DEGREES TO RADIANS	CONVERTING RADIANS TO DEGREES
Multiply the number of degrees by $\left(\frac{\pi \text{ radians}}{180°}\right)$.	Multiply the number of radians by $\left(\frac{180°}{\pi \text{ radians}}\right)$.

$\theta \cdot \frac{\pi}{180°}$ $\theta \cdot \frac{180°}{\pi}$

(A)

Degree measure	Radian measure
20°	$\frac{\pi}{180°} \cdot 20° =$ ☐
315°	☐ $\cdot 315° =$ ☐
600°	
−60°	
−540°	

(B)

Radian measure	Degree measure
$\frac{\pi}{8}$	$\frac{180°}{\pi} \cdot \frac{\pi}{8} =$ ☐
$\frac{4\pi}{3}$	☐ $\cdot \frac{4\pi}{3} =$ ☐
$\frac{9\pi}{2}$	
$-\frac{7\pi}{12}$	
$-\frac{13\pi}{6}$	

$\frac{\pi}{8} \cdot \frac{180°}{\pi} = \frac{180° \pi}{8\pi} = 22.5°$

$\frac{4\pi}{3} \cdot \frac{180°}{\pi} = \frac{720°}{3} = 240°$

$-\frac{7\pi}{12} \cdot \frac{180°}{\pi} = -105°$

$\frac{20^{\circ}}{1} \cdot \frac{\pi}{180^{\circ}} = \frac{20\pi}{180} = \boxed{\frac{\pi}{9}}$

$\frac{315^{\circ}}{1} \cdot \frac{\pi}{180^{\circ}} = \boxed{\frac{315\pi}{180}} = \boxed{\frac{7\pi}{4}}$

$\frac{-60^{\circ}}{} \cdot \frac{\pi}{180^{\circ}} = \frac{-60\pi}{180} = \boxed{\frac{-\pi}{3}}$

Your Turn

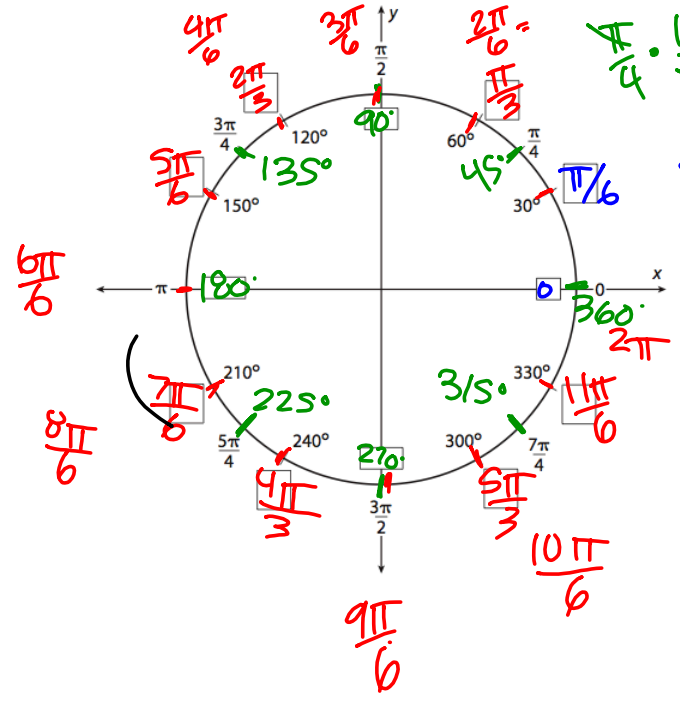
Convert each measure from degrees to radians or from radians to degrees.

8. -495° $\frac{\pi}{180} \quad \frac{-495}{1} \quad \frac{-495\pi}{180} \quad \boxed{\frac{-11\pi}{4}}$

9. $\frac{13\pi}{12}$

$\frac{13\pi}{12} \cdot \frac{180^{\circ}}{\pi} = \frac{2340^{\circ}}{12} = \boxed{195^{\circ}}$

7. The unit circle below shows the measures of angles of rotation that are commonly used in trigonometry, with radian measures outside the circle and degree measures inside the circle. Provide the missing measures.



pos: $\theta + 2\pi$

neg: $\theta - 2\pi$

For each angle, find the nearest two positive coterminal angles and the nearest two negative coterminal angles.

$-\frac{\pi}{2}$ $\quad \frac{-\pi}{2}+\frac{2\pi}{1}\cdot\frac{2}{2}$ $\quad \frac{11\pi}{6}$

$\frac{-\pi}{2}+\frac{4\pi}{2}=\frac{3\pi}{2}$

$-\frac{\pi}{2}-\frac{2\pi}{1}\cdot\frac{2}{2}$ $\quad -\frac{\pi}{2}-\frac{4\pi}{2}=\frac{-5\pi}{2}$

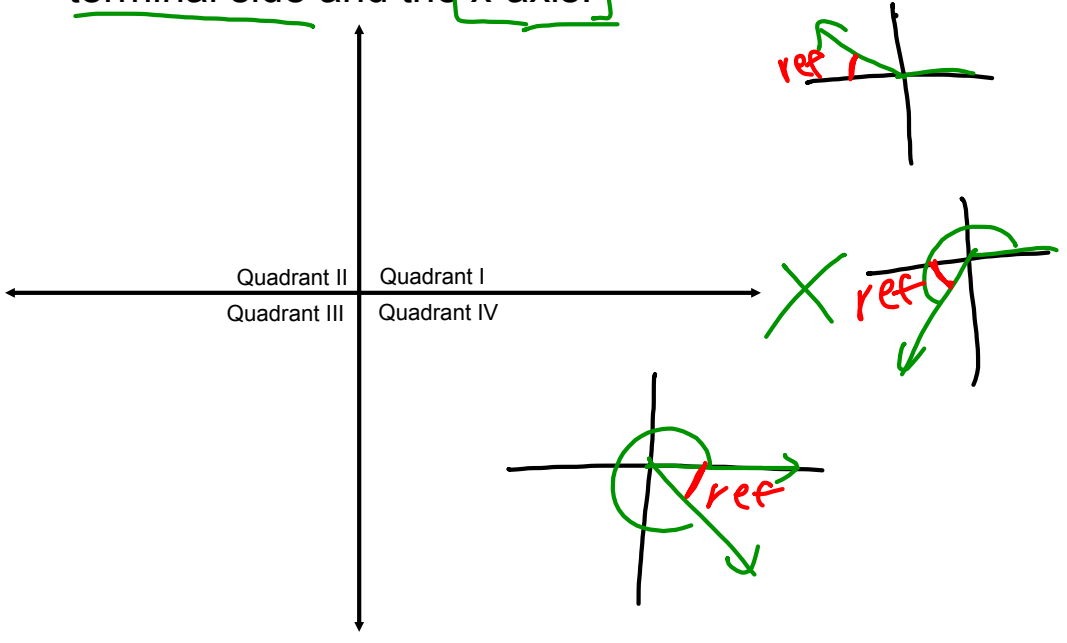
You Try:

$\frac{2\pi}{3}$ $\quad$ pos: $\frac{2\pi}{3}+\frac{2\pi}{1}\cdot\frac{3}{3}$ $\quad -\frac{\pi}{4}$

$\frac{2\pi}{3}+\frac{6\pi}{3}=\frac{8\pi}{3}$

neg: $\frac{2\pi}{3}-\frac{6\pi}{3}=\frac{-4\pi}{3}$

Reference Angles: The acute angle formed by the terminal side and the x-axis.

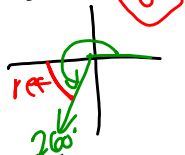


Given the angle, find the reference angle:

330° $360^\circ - 330^\circ$ 30°

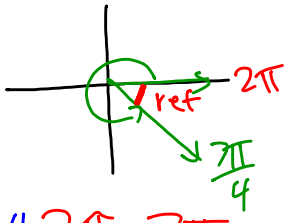
115° $180^\circ - 115^\circ$ 65°

-260° $-260^\circ - (-180^\circ)$ -80°



$\frac{2\pi}{3}$ $\pi - \frac{2\pi}{3}$ $\frac{\pi}{3}$

$\frac{7\pi}{4}$ $2\pi - \frac{7\pi}{4}$ $\frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$



$-\frac{11\pi}{6}$

You try! Given the angle, find the reference angle:

162°

$\frac{5\pi}{4}$

Reminder:

Angular velocity = $\frac{\theta}{t}$

Arclength:

given in degree: $s = \frac{\theta}{360} \cdot 2\pi r$

given in radians: $s = r\theta$

11. Astronomy A neutron star (an incredibly dense collapsed star) in the Sagittarius Galaxy has a radius of 10 miles and completes a full revolution every 0.0014 seconds. Find the angular velocity of the star in radians per second, then use this velocity to determine how far a point on the equator of the star travels each second. How does this compare to the speed of light (about 186,000 mi/sec)?

$AV = \frac{\theta}{t} = \frac{2\pi}{0.0014} = 4488$ rad/sec

$C = 2\pi r$
$= 2\pi 10$
$= 20\pi$

$\frac{20\pi}{.0014} = 44880$ mi/sec

$\frac{44880}{186000} = .24 = \boxed{24\%}$

12. Geography The northeastern corner of Maine is due north of the southern tip of South America in Chile. The difference in latitude between the locations is 103°. Using both degree measure and radian measure, and a north-south circumference of Earth of 24,860 miles, find the distance between the two locations.

difference

$$\frac{103^\circ}{360^\circ} = \frac{x}{24860}$$

Earth

$x = 7113 \text{ mi}$