Term 2 Review
Secondary III
Units 4, 5, and 6

Name: KEY
Date: ______________ Class: __________

Unit 4: Complex Zeros of Polynomials

Perform the indicated operation and write the result in standard form

1. $(1+i)^2$ $(1+i)(1+i)$

$1+i+i+\cancel{i^2}$ -1

$\boxed{2i}$

2. $\sqrt{-75}$ $\boxed{5i\sqrt{3}}$

$-3\ 25$

$5\ 5$

3. $(3-2i)+(-2+5i)$

$\boxed{1+3i}$

4. $(5-7i)-(3-2i)$

$\boxed{2-5i}$

Solve the following polynomials

5. $x^2-6x+13=0$

$x=\frac{6\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$ $=\frac{6\pm 4i}{2}$

$=\frac{6\pm\sqrt{36-52}}{2}$ $\boxed{x=3\pm 2i}$

$=\frac{6\pm\sqrt{-16}}{2}$

6. $x^2+24=0$ $\boxed{x=\pm 2i\sqrt{6}}$

$-24\quad -24$

$\sqrt{x^2}=\sqrt{-24}$

$2\ 12$

$2\ 6$

$2\ 3$

State how many complex and real zeros the function has

7. $f(x)=x^4-2x^2+3x-4$

Complex: 4

Real: 2

8. $f(x)=x^5-2x^2-3x+6$

Complex: 5

Real: 1

Write a polynomial in **factored form** with minimum degree given the following zeros.

9. $3, 3+2i$, $3-2i$

$f(x)=(x-3)(x-(3+2i))(x-(3-2i))$

Write a polynomial in **standard form** with minimum degree given the following zeros.

10. $1+2i, 1-2i$

$f(x)=(x-(1+2i))(x-(1-2i))$

Find all the zeros and write a linear factorization of the function.

11. $f(x)=x^3-10x^2+44x-69$ $\frac{\pm 1,3,23}{1}$

$3\,|\ 1\ \ -10\ \ 44\ \ -69$
$\downarrow\ \ 3\ \ -21\ \ 69$
$1\ \ -7\ \ 23\ \ \boxed{0}$

$(x^2-7x+23)(x-3)$

$\frac{7\pm\sqrt{49-92}}{2}$

$\frac{7\pm\sqrt{-43}}{2}$

$f(x)=(x-3)\left(x-\left(\frac{7+i\sqrt{43}}{2}\right)\right)\left(x-\left(\frac{7-i\sqrt{43}}{2}\right)\right)$

12. $f(x)=x^4+x^3+5x^2-x-6$ $\frac{\pm 1,2,3,6}{1}$

$1\,|\ 1\ \ 1\ \ 5\ \ -1\ \ -6$
$\downarrow\ \ 1\ \ 2\ \ 7\ \ 6$
$-1\,|\ 1\ \ 2\ \ 7\ \ 6\ \ \boxed{0}$
$\downarrow\ \ -1\ \ -1\ \ -6$
$1\ \ 1\ \ 6\ \ \boxed{0}$

$(x-1)(x+1)(x^2+x+6)$

$\frac{-1\pm\sqrt{1-24}}{2}$

$\frac{-1\pm\sqrt{-23}}{2}$ $\frac{-1\pm i\sqrt{23}}{2}$

$f(x)=(x-1)(x+1)\left(x-\left(\frac{-1+i\sqrt{23}}{2}\right)\right)\left(x-\left(\frac{-1-i\sqrt{23}}{2}\right)\right)$

Use the given zero to find all the zeros of the function.

13. $4i,\ f(x)=x^4+13x^2-48$

$4i\,|\ 1\ \ 0\ \ 13\ \ 0\ \ -48$
$\downarrow\ \ 4i\ \ -16\ \ -12i\ \ 48$
$1\ \ 4i\ \ -3\ \ -12i\ \ \boxed{0}$

$-4i\,|\ 1\ \ 4i\ \ -3\ \ -12i$
$\downarrow\ \ -4i\ \ 0\ \ 12i$
$1\ \ 0\ \ -3\ \ \boxed{0}$

$x^2-3=0$
$+3\ \ +3$
$\sqrt{x^2}=\sqrt{3}$

$\boxed{\text{Zeros: } x=4i\ \ x=\sqrt{3}\ \ x=-4i\ \ x=-\sqrt{3}}$

$(3-2i)(-3-2i)$
$9-6i+6i+4i^2$
-4
$(3-2i)(6+4i)$
$18+12i-12i-8i^2$
$+8$

14. $3-2i,\ f(x)=x^4-6x^3+11x^2+12x-26$

$3-2i\,|\ 1\ \ -6\ \ 11\ \ 12\ \ -26$
$\downarrow\ \ 3-2i\ \ -13\ \ -6-4i\ \ 26$
$1\ \ -3-2i\ \ -2\ \ 6+4i\ \ \boxed{0}$

$3+2i\,|\ 1\ \ -3-2i\ \ -2\ \ 6+4i$
$\downarrow\ \ 3+2i\ \ 0\ \ -6-4i$
$1\ \ 0\ \ -2\ \ \boxed{0}$

$x^2-2=0$
$+2\ \ +2$
$\sqrt{x^2}=\sqrt{2}$

$\boxed{\text{Zeros: } x=3-2i\ \ x=\sqrt{2}\ \ x=3+2i\ \ x=-\sqrt{2}}$

Unit 5: Rational expressions and equations

Solve algebraically the following rational equations

1.

$(x-1)(x-3)\cdot\frac{2x}{x-1}+\frac{1}{x-3}\cdot(x-1)(x-3)=\frac{2}{(x-1)(x-3)}\cdot(x-1)(x-3)$

$x\neq 1,3$

$2x(x-3)+1(x-1)=2$

$2x^2-6x+x-1=2$
$-2\ \ -2$

$2x^2-5x-3=0$

$x=\frac{5\pm\sqrt{25+24}}{4}$

$x=\frac{5\pm\sqrt{49}}{4}=\frac{5\pm 7}{4}$

$x=\cancel{3},\ -\frac{1}{2}$

$\boxed{x=-\frac{1}{2}}$

2.

$\frac{6(x-2)}{2}\cdot\frac{x+1}{3x-6}=\frac{5x}{6}\cdot 6(x-2)+\frac{1}{x-2}\cdot 6(x-2)$

$3(x-2)$

$x\neq 2$

$2(x+1)=5x(x-2)+1(6)$

$2x+2=5x^2-10x+6$
$-2x\ \ -2\ \ \ \ -2x\ \ -2$

$0=5x^2-12x+4$

$x=\frac{12\pm\sqrt{144-80}}{2(5)}$

$=\frac{12\pm\sqrt{64}}{10}$

$=\frac{12\pm 8}{10}$ $\frac{20}{10},\ \frac{4}{10}$

$x=\cancel{2},\ \frac{2}{5}$

$\boxed{x=\frac{2}{5}}$

Multiply or divide the following rational expressions and find the excluded vales.

3. Multiply $\frac{3x+6}{x+2}\bullet\frac{x-3}{x-4}$ $\quad x \neq -2, 4$

4. Divide $\frac{x+3}{x+2}\div\frac{x^2+3x}{2x-4}$ $\quad x \neq -2, 0, -3, 2$

$\frac{(3x+6)(x-3)}{(x+2)(x-4)} = \frac{3\cancel{(x+2)}(x-3)}{\cancel{(x+2)}(x-4)}$

$\boxed{= \frac{3(x-3)}{(x-4)}}$

$\frac{x+3}{x+2}\cdot\frac{2x-4}{x^2+3x}$

$= \frac{\cancel{x+3}}{x+2}\cdot\frac{2(x-2)}{x\cancel{(x+3)}}$

$\boxed{= \frac{2(x-2)}{x(x+2)}}$

Add or subtract the following expressions, simplify the results, and note the excluded values.

5. $\frac{4}{\cancel{x^2-x}} - \frac{x+2}{x-1}\cdot\frac{x}{x}$

$x(x-1)$

$\frac{4}{x(x-1)} - \frac{x(x+2)}{x(x-1)}$ $\quad x^2+2x$

$= \frac{4-x^2-2x}{x(x-1)}$

$\boxed{= \frac{-x^2-2x+4}{x(x-1)}}$

$x = 0, 1$

6. $\frac{x}{x}\cdot\frac{1}{3+x}+\frac{3-x}{x}\cdot\frac{(3+x)}{(3+x)}$

$\frac{x}{x(3+x)} + \frac{(3-x)(3+x)}{x(3+x)} = \frac{x+}{x(3+x)} \quad \frac{9 \mp x^2}{x(3+x)} = \boxed{\frac{-x^2+x+9}{x(3+x)}}$

$x \neq 0, -3$

Find the LCD of the following rational equations:

7. $\frac{5}{(x-2)(x-1)}-\frac{1}{x-2}=0$

$\boxed{\text{LCD} = (x-2)(x-1)}$

8. $\frac{x+2}{x}-\frac{4}{x-1}+\frac{2}{x^2-x}=0$

$x(x-1)$

$\boxed{\text{LCD}: x(x-1)}$

9. A restaurant has two pastry ovens. When both ovens are used, it takes about 3 hours to bake the bread needed for one day. When only the large oven is used, it takes about 4 hours to bake the bread for one day. About how long would it take to bake the bread for one day if only the small oven were used? Explain how you got your answer.

$12s\cdot\frac{1}{4} + 12s\cdot\frac{1}{s} = 12s\cdot\frac{1}{3}$

$\boxed{s = 12 \text{ hours}}$

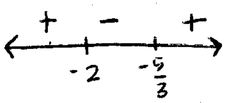
$3s + 12 = 4s$

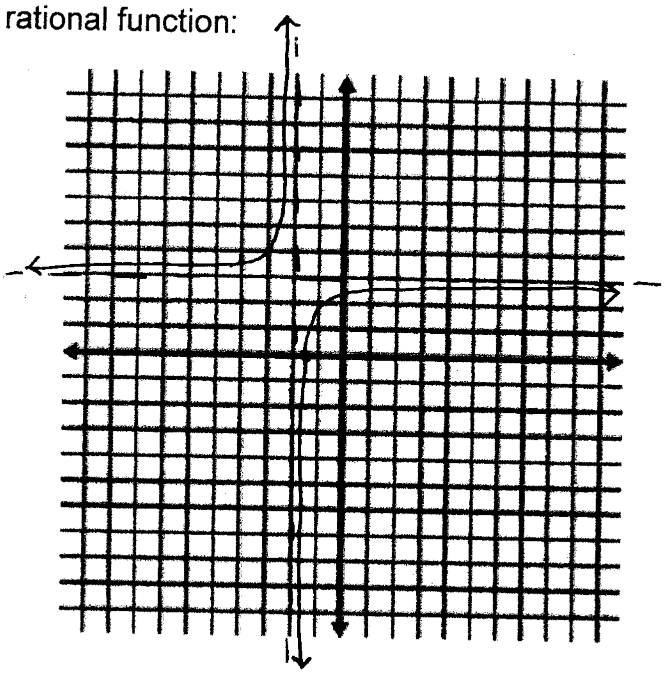
$-3s \qquad -3s$

Unit 6: Rational Graphs

Find the following information and graph each rational function:

1. $f(x)=\dfrac{3x+5}{x+2}$

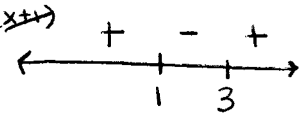


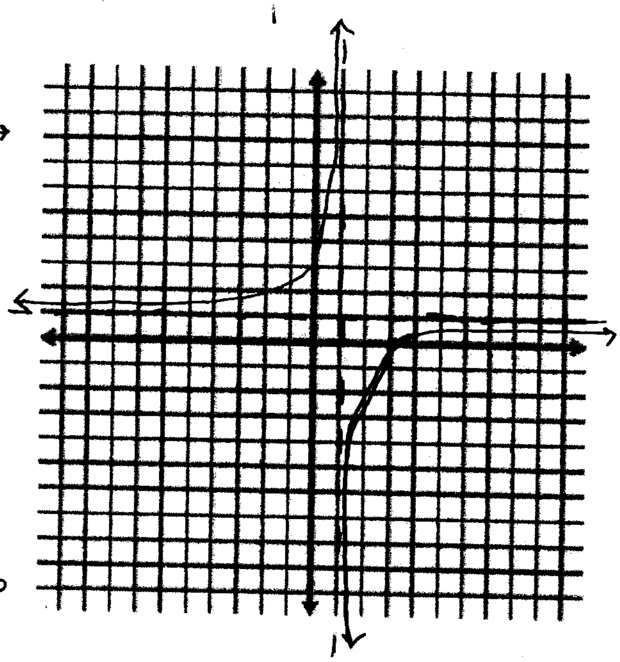


Domain: $(-\infty,-2)\cup(-2,\infty)$
Range: $(-\infty,3)\cup(3,\infty)$
x- int: $(-\frac{5}{3},0)$
Vertical Asymptote: $x=-2$
Horizontal Asymptote: $y=3$
Increasing: $(-\infty,-2)\cup(-2,\infty)$
Decreasing: Never
End Behavior: $\lim_{x\to-\infty} f(x)=3$ $\lim_{x\to\infty} f(x)=3$
Asymptote Behavior:

$\lim_{x\to-2^-} f(x)=\infty$ $\lim_{x\to-2^+} f(x)=-\infty$

2. $f(x)=\dfrac{x^2-2x-3}{x^2-1}$ $\dfrac{(x-3)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$





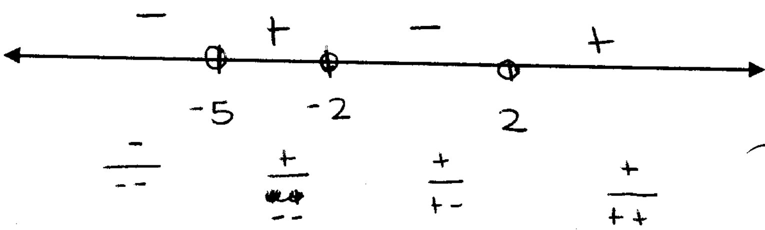
Domain: $(-\infty,1)\cup(1,\infty)$
Range: $(-\infty,1)\cup(1,\infty)$
x- int: $(3,0)$
Vertical Asymptote: $x=1$
Horizontal Asymptote: $y=1$
Increasing: $(-\infty,1)\cup(1,\infty)$
Decreasing: Never
End Behavior: $\lim_{x\to-\infty} f(x)=1$ $\lim_{x\to\infty} f(x)=1$
Asymptote Behavior

$\lim_{x\to1^-} f(x)=\infty$ $\lim_{x\to1^+} f(x)=-\infty$

Solve the following inequalities using a sign chart:

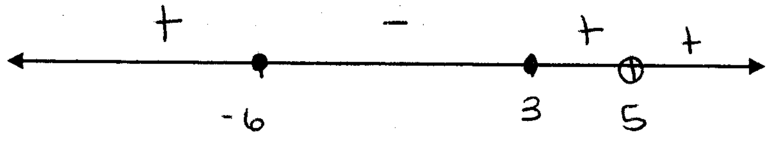
3. $\dfrac{x+5}{x^2-4}<0$

$\dfrac{x+5}{(x+2)(x-2)}<0$



$\boxed{(-\infty,-5)\cup(-2,2)}$

4. $\dfrac{x^2+3x-18}{x^2-10x+25} \geq 0$



$\dfrac{(x+6)(x-3)}{(x-5)(x-5)} \geq 0$

$\frac{--}{--}$ $\frac{+-}{--}$ $\frac{++}{--}$ $\frac{++}{++}$

$\boxed{(-\infty,-6] \cup [3,5) \cup (5,\infty)}$

Given the following functions, find all holes, asymptotes, and intercepts.

5. $f(x) = \dfrac{x-3}{x^2+6x+5}$

$\dfrac{x-3}{(x+5)(x+1)}$

Holes: None
VA: $x=-5$, $x=-1$
HA: $y=0$
x-int: $(3,0)$
y-int: $(0,-\frac{3}{5})$

6. $f(x) = \dfrac{x(x+4)^2\cancel{(x-5)}}{(x-5)^{\cancel{2}}(x+1)^2}$

$\dfrac{x(x+4)^2}{(x-5)(x+1)^2}$

Holes: None
VA: $x=5$
HA: $x=1$
x-int: $(0,0)$, $(-4,0)$
y-int: $(0,0)$

Describe how the graph of *g(x)* is related to the graph $f(x)=\dfrac{1}{x}$.

7. $g(x)=\dfrac{5}{x}-3$

Vertical Stretch of 5
Shifted Down 3

8. $g(x)=\dfrac{-1}{x}+5$

Flipped over x-axis
Shifted Up 5

9. $g(x)=-\dfrac{1}{(x-2)}+4$

Shifted Right 2
Shifted Up 4
Flipped over x-axis

10. How do you find the asymptotes (vertical and end behavior) of a rational function?

Vertical Asymptotes: Set the denominator equal to 0

End behavior:

11. A basketball team has won 16 games out of 23 games played, for a winning percentage (expressed as a decimal) of $\frac{16}{23} \approx 0.696$. How many consecutive games must the team win to raise its winning percentage to 0.750?